IMPORTANT EQUATIONS

1: THE PARALLAX-DISTANCE EQUATION (Lecture 2)

In symbols:

$$d = 1/p.$$

d = distance in parsecs
p = parallax in arc-seconds

In words: The distance of a star in parsecs is one over its parallax in arc-seconds.

Example:

An astronomer at the U.S. Naval Observatory measures the parallax of a star to be 0.1 arc-seconds. What is the distance of the star in parsecs? How long did light from the star take to reach the earth?

The distance is $1/0.1 = 10$ parsecs. One parsec is 3.26 light-years, so light from the star took $3.26 \times 10 = 32.6$ years to get to earth.

2: THE FLUX-LUMINOSITY-DISTANCE EQUATION (Lecture 2)

In symbols:

$$f = \frac{L}{4\pi d^2}.$$

L = intrinsic luminosity of the source [ergs/second]
d = distance of the source [cm]
f = apparent brightness (flux) of the source [ergs/s/cm^2]

In words: The apparent brightness of a source is equal to the intrinsic luminosity divided by 4π times the square of the distance.

We can interpret this equation by thinking of the photons being "spread out" over a sphere whose area is $4\pi d^2$ (see Figure 5-7).

Examples:

(1) Star B has twice the intrinsic luminosity of star A, but it is twice as far away. What is its apparent brightness relative to that of star A?

The apparent brightness is proportional to the luminosity and to the inverse square of the distance, so star B appears $2/2^2 = 1/2$ as bright as star A.

More formally:

$$\frac{f_B}{f_A} = \frac{L_B/4\pi d_B^2}{L_A/4\pi d_A^2} = \frac{L_B}{L_A}\frac{4\pi d_A^2}{4\pi d_B^2} = \frac{1}{2}.$$

(2) Stars A and B have the same intrinsic luminosity, but the apparent brightness of star A is 100 times greater. How much further away is star B?

Since the stars have the same luminosity, apparent brightness is just proportional to the inverse square of the distance. Star B will be 100 times fainter if it is 10 times further away.

More formally:

$$\frac{d_B^2}{d_A^2} = \frac{L_B/4\pi f_B}{L_A/4\pi f_A}$$

so

$$\frac{d_B}{d_A} = \sqrt{\frac{f_A}{f_B}} = 10.$$

3: THE LUMINOSITY OF A SPHERICAL BLACKBODY (Lecture 5)

In symbols:

$$L = (4\pi\sigma)R^2T^4.$$

L = luminosity of a spherical blackbody [ergs/second]
R = radius of the body [cm]
T = temperature of the body (degrees Kelvin)
σ = the Stefan-Boltzmann constant

In words: The luminosity of a spherical blackbody is proportional to the square of its radius and to the fourth power of its temperature. The constant of proportionality is $4\pi\sigma$.

Note: A star is not a perfect blackbody, but it is close. Astronomers define the *effective temperature* of a star, T_e, by the equation

$$L_* = (4\pi\sigma)R_*^2T_e^4.$$

Examples:

(1) Star A and star B have the same radius, but star B is twice as hot. How much more luminous is star B?

The luminosity is proportional to T^4, so star B is $2^4 = 16$ times more luminous.

More formally,

$$\frac{L_B}{L_A} = \frac{4\pi\sigma R_B^2T_B^4}{4\pi\sigma R_A^2T_A^4} = \frac{T_B^4}{T_A^4} = 16.$$

(2) Two stars have the same spectral type, and they have the same apparent brightness (flux). However, star A has a parallax of 1", and star B has a parallax of 0.1". How big is star B relative to star A?

We have to pull together several different equations here. The distance to a star is $d = 1/p$, where p is the parallax, so star B is 10 times more distant than star A. The apparent flux of a star is $f = L/(4\pi d^2)$, so if the two stars have the same apparent flux, star B must be 100 times more luminous. Since the two stars have the same spectral type, they are the same temperature. But $L \propto R^2T^4$, so if T is the same and star B is 100 times more luminous, it must be ten times bigger than star A.

4: THE EQUATION OF STATE FOR AN IDEAL GAS (Lecture 6)

In symbols:

$$P = knT.$$

P = pressure of the gas [dynes/cm^2]
n = number density [atoms/cm^3]
T = temperature [degrees Kelvin]
k = Boltzmann's constant

In words: The pressure exerted by an ideal gas is proportional to its density and to its temperature in degrees Kelvin. The constant of proportionality is Boltzmann's constant k, which tells how much energy a typical atom has when the temperature is T.

Examples:

(1) I have some high-pressure oxygen stored in a tank. I siphon half of the oxygen into another tank. If the temperature does not change when I do this, what happens to the gas pressure?

The volume of the tank stays the same, but I reduce the amount of gas by a factor of two, so the density n goes down by a factor of two. Since T doesn't change, the pressure drops by a factor of two.

(2) I have some helium gas in a jar. I heat this jar over a burner so that the temperature of the helium (in degrees Kelvin) doubles. What happens to the pressure?

The density doesn't change, the temperature doubles, so the pressure doubles.

5: THE CENTRAL TEMPERATURE OF A STAR (Lecture 6)

In symbols:

$$T_c = \left(\frac{G}{k}\right)\frac{M m_a}{R_{\rm avg}}$$

T_c = central temperature of star [degrees Kelvin]
M = mass of star [grams]
m_a = average mass of atom [grams]
$R_{\rm avg}$ = "average" radius of material in star [cm]
G = Newton's gravitational constant
k = Boltzmann's constant

In words: The central temperature of a star is proportional to the mass of the star times the mass of typical atom divided by the "average" radius of the star. The constant of proportionality is Newton's gravitational constant G divided by Boltzmann's constant k.

Comment: This equation was derived (approximately) in Lecture 6. It only holds true for stars in which the central pressure is determined by the ideal gas equation.

Examples:

(1) Star A is twice as massive as star B, but it is also twice as big. Which star has the higher central temperature?

Doubling the mass is canceled by doubling the radius; the stars have the same central temperature.

(2) Stars A and B are the same mass and the same radius, but star A is made entirely of hydrogen and star B entirely of helium. Which star has the higher central temperature?

The mass of a helium atom is about four times the mass of a hydrogen atom. If M and R are the same but m_a is four times higher, the central temperature of star B is four times higher than that of star A.

An implication: If a star becomes more centrally concentrated, its central temperature goes up because $R_{\rm avg}$ goes down, even if the radius of the surface doesn't change.

6: THE MASS – ROTATION SPEED EQUATION (Lecture 18)

In symbols:

$$M_{\rm int}(R) = \frac{V_{\rm rot}^2 R}{G}.$$

$M_{\rm int}(R)$ = mass interior to radius R [solar masses]
$V_{\rm rot}$ = rotation velocity [km/s]
R = radius (distance from center of galaxy) [kiloparsecs]
G = Newton's gravitational constant

In words: The mass interior to a radius R in a disk galaxy is equal to the square of the rotation speed at this radius multiplied by the radius and divided by Newton's gravitational constant.

Comments:

This equation can be derived from Newton's law of gravity and law of motion. A similar equation can be used to describe the solar system or other gravitating systems with circular rotation.

Examples:

(1) Two galaxies are the same radius, but one of them is four times more massive than the other. How much higher is its rotation speed?

Since $M \propto V_{\rm rot}^2$ when the radius is the same, the more massive galaxy has double the rotation speed.

(2) When I was in graduate school, I used a radio telescope in New Mexico to observe hydrogen gas in a galaxy called UGC 12591, which is the most rapidly rotating spiral galaxy known. My observations detected gas with a rotation speed of 465 km/s at a distance of 20 kpc from the center of the galaxy, and by plugging these numbers (and the value of G) into the equation, I could tell that the inner 20 kpc of the galaxy contained a mass of 1 trillion solar masses. My observations also detected gas 36 kpc from the center, and it also had a rotation speed of 465 km/s. How much mass is contained in the inner 36 kpc of UGC 12591?

Since the rotation speed at 20 kpc and 36 kpc is the same but the radius at 36 kpc is 36/20=1.8 times bigger, there must be 1.8 trillion solar masses in the inner 36 kpc.

7: HUBBLE'S LAW (Lecture 23)

In symbols:

$$v = Hd.$$

v = recession velocity, measured from Doppler shift [km/s]
d = distance [Mpc]
H = Hubble's constant [km/s/Mpc]

In words: A galaxy's recession velocity, which can be measured from its Doppler shift, is proportional to its distance. The constant of proportionality is Hubble's constant H.

Comments:

- Hubble's law is an empirical relation, discovered through observation, and it is not exact. It has a natural interpretation within the Big Bang theory.
- Hubble's constant is difficult to measure. The best current estimate is $H \approx 70$ km/s/Mpc, but within the observational uncertainties, H could be as high as 90 or as low as 50. A better measurement of H is a major goal of observational cosmology.

Examples:

(1) What is the recession velocity of a galaxy 10 Mpc away, assuming $H = 70$ km/s/Mpc?

The velocity is $v = Hd = (70 \text{ km/s/Mpc}) \times (10 \text{ Mpc}) = 700$ km/s.

(2) By measuring a galaxy's Doppler shift, I determine that it is receding from the earth at 7000 km/s. How distant is the galaxy, assuming $H = 70$ km/s/Mpc?

The distance is $d = v/H = (7000 \text{ km/s})/(70 \text{ km/s/Mpc}) = 100$ Mpc.