RELATIVITY MTH6132

PROBLEM SET 9

Please write your name and student number at the top of your assignment and staple all the pages together.

1*. Derive the timelike geodesic equations for the Schwarzschild metric.

2. Show that the radial timelike geodesics for the Schwarzschild satisfy

$$\dot{r}^2 = \left(\frac{dr}{d\tau}\right)^2 = l^2 - 1 + \frac{2GM}{r}$$

where $l = [1 - (2GM/r)]\,\dot{t}$. Assuming $l = 1$, find the finite proper time it takes a freely falling particle to fall from $r = 2GM$ to $r = 0$.

3*. A proper clock (one that keeps the proper time τ) is on a satellite orbiting a central mass M in a circle of Schwarzschild radius D (i.e. in a circular orbit of radius $r = D$). Show that the time to complete one revolution as measured by the clock is

$$\tau = 2\pi \left(\frac{D^3}{GM}\right)^{1/2} \left(1 - \frac{3GM}{D}\right)^{1/2}$$

Hint: write down the timelike geodesic equations for the Schwarzschild metric. For a circular orbit, take $\theta = \pi/2 =$ constant and $r = D =$ constant. Now, using one of the resulting equations $D^2\dot{\phi} = h =$ constant, the proper time τ can be found by integrating this expression to obtain $\tau = 2\pi D^2/h$. Use the other equations to eliminate h.

To be handed in by Wednesday 14th December, 6pm.

Dr. Juan A. Valiente Kroon (G56)