RELATIVITY MTH6132

PROBLEM SET 8

(1*) The metric for a particular two–dimensional Riemannian spacetime is given by

$$ds^2 = -e^{2Ar}dt^2 + dr^2$$

where A is an arbitrary constant. Employ the geodesic equation (Euler Lagrange Eqs) to calculate all the components of the connection $\Gamma^a{}_{bc}$ for this metric. Hence calculate the R_{tt} component of the Ricci tensor. Can this metric represent a flat space?

You may assume that the Ricci tensor is defined in terms of the connection as

$$R_{ab} = -\partial_b \Gamma^c{}_{ca} + \partial_c \Gamma^c{}_{ab} - \Gamma^c{}_{da}\Gamma^d{}_{cb} + \Gamma^c{}_{cd}\Gamma^d{}_{ab}.$$

(2*) (i) From the formula for the Christoffel symbols

$$\Gamma^a_{bc} = \frac{1}{2} g^{ae}(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc})$$

show that $\Gamma_{abc} \equiv g_{af}\Gamma^f_{bc}$ is given by

$$\Gamma_{abc} = \frac{1}{2}(\partial_b g_{ac} + \partial_c g_{ba} - \partial_a g_{bc}).$$

(ii) The Riemann curvature tensor of a certain manifold is of the form

$$R_{abcd} = K\left(g_{ac}g_{bd} - g_{ad}g_{cb}\right),$$

with K a constant. Show that:

- The tensor R_{abcd} so defined has the symmetries of the Riemann tensor discussed in the lecture;
- Observing that $\nabla_a g_{bc} = 0$, show that $\nabla_e R_{abcd} = 0$. What sort of surface could have a constant curvature?
- Sow that the corresponding Ricci tensor is proportional to the metric, and that the Ricci scalar is a constant.

(3*) (i) Using the formula of the Riemann tensor in locally inertial coordinates show that

$$R_{abcd} + R_{acdb} + R_{adbc} = 0.$$

(ii) Using the symmetries of the Riemann tensor show that the above identity is equivalent to

$$R_{a(bcd)} = 0.$$

To be placed in the BLUE BOX on 2nd floor of the Maths building by Wednesday 7 December, 6pm.

Dr. Juan A. Valiente Kroon (G56)