RELATIVITY MTH6132

PROBLEM SET 7

1. Starting from the Minkowski line element in Cartesian coordinates

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2,$$

show that in spherical coordinates

$$x = r\sin\theta\sin\varphi, \quad y = r\sin\theta\cos\varphi, \quad z = r\cos\theta,$$

the line element is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2.$$

2.* The metric for a particular two-dimensional Lorentzian manifold is given by

$$ds^2 = -y^3dx^2 + x^4dy^2$$

Employ the geodesic equation (Euler-Lagrange equations) to calculate all the components of the connection $\Gamma^a{}_{bc}$ for this metric. Use this result to calculate the $R^x{}_{yxy}$ component of the Riemann tensor.

3*. Show that any general (non-symmetric) covariant tensor of rank two, T_{ab} say, can be expressed as the sum of its symmetric part, $T_{(ab)}$, and anti-symmetric part, $T_{[ab]}$. Hence prove that

$$g^{ab}T_{ab} = g^{ab}T_{(ab)},$$

where g^{ab} is a general Riemannian metric tensor.

4.* Explain what is meant by a Local Inertial Frame. Show that in such a frame the Riemann tensor can be expressed in the form

$$R_{abcd} = \frac{1}{2}\left(\partial_d\partial_a g_{bc} + \partial_c\partial_b g_{ad} - \partial_c\partial_a g_{bd} - \partial_d\partial_b g_{ac}\right)$$

at a specific point p. Employ this expression to show that in such a frame

$$R_{abcd} = -R_{bacd}$$

at p. Is this relation valid in an arbitrary frame of reference? Explain your reasoning.

5. Write down the definition of the Ricci tensor, R_{bd}, and Ricci scalar in terms of the contractions of the Riemann tensor, $R^a{}_{bcd}$. Using the symmetries of the curvature tensor prove that R_{bd} is symmetric.

To be placed in the BLUE BOX on 2nd floor of the Maths building by 6pm Wednesday 30 November 2010.

Dr. Juan A. Valiente Kroon (G56)