

RELATIVITY MTH6132

PROBLEM SET 5

HAND IN ONLY THE STARRED QUESTIONS.

- 1*** (i) Write down the transformation laws under general coordinate transformations for a tensor of type $(0, 1)$ and a tensor of type $(2, 1)$ respectively.
(ii) Show that the product of these tensors is a tensor of type $(2, 2)$.
(iii) Prove that the contraction of a tensor of type $(2, 2)$ results in a tensor of type $(1, 1)$.

2 Show that the product $A_i B^i$ formed from a covariant vector A_i and a contravariant vector B^i is a scalar. [**Hint:** Show that $A_i B^i = A'_i B'^i$].

3* If the quantity $T^{ab} V_a W_b$ is a scalar, where V_a and W_b are arbitrary covariant vectors, show that the 2-indexed symbol T^{ab} is a rank $(2, 0)$ tensor.

5 Show that if the 2-indexed geometric symbol A^{ij} is symmetric and the 2-indexed symbol B_{ij} is antisymmetric, then

$$A^{ij} B_{ij} = 0$$

6* Prove that if the equation $V_{ab} = V_{ba}$ is true in a given coordinate frame, it is true in all frames (i.e. prove that it has a tensorial property).

To be placed in the BLUE BOX on 2nd floor of the Maths building by Wednesday 16th November.

Dr. Juan A. Valiente Kroon (G56)