RELATIVITY (MTH6132)

PROBLEM SET 1

HAND IN ONLY the STARRED QUESTIONS.

Write your name and student number at the top of your assignment and staple all the pages together.

1 Starting from the Galilean transformation in the form

$$\underline{r}' = \underline{r} - \underline{v}t, \quad \underline{v} = (v_x, v_y, v_z),$$

show that the scalar wave equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

does not remain invariant under these transformations.
[**Hint**: recall that $t = t(t', x', y', z')$ and $x = x(t', x', y', z')$. Use the chain rule of partial differentiation to show that

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - v_x\frac{\partial}{\partial x'} - v_y\frac{\partial}{\partial y'} - v_z\frac{\partial}{\partial z'} = \frac{\partial}{\partial t'} - \underline{v}\cdot\nabla'$$

and similar formulae for $\frac{\partial}{\partial x}$ etc.]

2 Starting from the Lorentz transformations between two frames F and F' in standard configuration (with F' moving with velocity of magnitude v relative to F) show by adding and subtracting x' and ct' that

$$ct' - x' = \epsilon(ct - x), \qquad ct' + x' = \frac{1}{\epsilon}(ct + x)$$

where $\epsilon = \sqrt{\frac{1+v/c}{1-v/c}}$. Use these expressions to show that the combination of two Lorentz transformations with velocities v_1 and v_2, respectively, is a Lorentz transformation. What is the velocity of the composite transformation?

3 In Joe's frame of reference a ray of light is shot at $t = 0$ from $x = L$ towards the origin where a mirror reflects it back. The ray reaches $x = 2L$ at time t_1. Draw a spacetime of the situation as seen by Joe. Draw also the situation as seen by Moe who is moving with positive velocity $v < c$ along Joe's x-axis.

To be handed in on Wednesday 12th October by 6pm in the blue box in the second floor of the School of Mathematical Sciences.

Dr. Juan A. Valiente Kroon (G56)

The following is not to be handed in.

1. Define the following:

- Frame of reference
- Inertial Frame
- Galilean Principle of Relativity
- Standard configuration
- Spacetime and worldlines

2. Give the Galilean transformations between inertial frames in standard configuration.

3. Show that Newton's second law is invariant under Galilean transformations.

4. State Einstein's postulates of Special Relativity.