Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 1

Monday, 29 January 2007

Time allowed for Examination: 3 hours

Answer **ALL** questions

Books and notes may be consulted

The paper is split into the following sections with their weights in brackets:

1. Symmetries and Conservation Laws (1/3)
2. Neutrino physics (2/9)
3. CP violation (1/9)
4. Hadron Colliders (1/6)
5. QCD phenomenology (1/12)
6. ILC physics (1/12)

Please answer different sections in different answer books as they will be marked by different people.

Section 1: Symmetries and Conservation Laws

Question 1 - SO(3)

(a) Explain what SO(3) is in terms of the matrix operators which define it and identify the corresponding transformations. **[1 mark]**

(b) Show how SO(3) satisfies the group axioms. **[4 marks]**

(c) The transformations associated with SO(3) are examples of isometries. Isometries preserve the scalar product (distance) between transformed vectors (i.e. the shape of an object). Identify two other classes of isometry from the Poincare set of transformations. Explain why they are not part of SO(3). **[2 marks]**

(d) Starting from the matrix representation of a typical SO(3) operator in a vector space of 3D vectors, derive the corresponding generator. **[2 marks]**

Question 2 - SU(3)

(a) What are the characteristic differences between SO(3) and SU(3)? How many generators does each group have? **[1 mark]**

(b) Show that $r\bar{r}$ is not a singlet of $\mathrm{SU}(3)_{colour}$, but that $r\bar{r} + b\bar{b} + g\bar{g}$ is.
Hint: Consider the infinitesimal transformations generated by λ_1, λ_2, λ_3 and λ_8 separately along with a suitable use of "likewise". Under a transformation U, $c_1c_2c_3 \to U(c_1)U(c_2)U(c_3)$ - where the operator acts separately on the three individual colour states. An anticolour (treated as a column vector) transforms like: $\bar{c} \to U^*\bar{c}$. The λ matrices are provided below.
(This question may take longer than others, so you may wish to do it at the end.)
[6 marks]

(c) Where would you expect to encounter the colour singlet description in the previous question? Where would you not expect to encounter it? **[1 mark]**

(d) List some of the consequences of "colour". **[1 mark]**

(e) Using Young Tableaux, identify the multiplets that can be obtained under SU(3) from the combination $3 \otimes \bar{3}$. **[1 mark]**

(f) Explain why the $\mathrm{SU}(3)_{flavour}$ singlet state $u\bar{u} + d\bar{d} + s\bar{s}$ is not seen. **[1 mark]**

The fundamental colour representations are:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and the SU(3) generators are:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Section 2: Neutrino physics

Question 1 [4 marks]

(a) The Homestake experiment has pioneered solar neutrino detection and measured the solar neutrino flux for about 30 years. Describe briefly the experiment and discuss the merit of radiochemical solar neutrino detection.

(b) The observed rate of interactions was 2.56 ± 0.23 SNU (1 SNU $= 10^{-36}$ interactions per target atom per second). Given an absorption cross-section on ^{37}Cl of $\sigma = 10^{-42} cm^2$, compute the measured flux.

(c) Given a predicted rate of 8.5 ± 1.8 SNU, compute the oscillation probability of an electron neutrino to a neutrino of different flavour. Assuming oscillation in vacuum between two neutrino species and $sin^2 2\theta = 0.86$, what value of Δm^2 does Homestake suggest? Compare this value with the current best fit to solar neutrino data. Is it compatible? Justify your answer.

Question 2 [6 marks]

(a) Describe the MINOS experiment and the main experimental results. Use the measured Δm^2_{23} and $sin^2 2\theta_{23}$ to estimate the probability of ν_μ survival [$\mathrm{P}(\nu_\mu \rightarrow \nu_\mu)$] at an average energy of 3 GeV. Can matter effects be neglected? Justify your answer.

(b) Giving that 215 events are observed in the far detector (Mass=5.4 kton, L=735km), derive the number of events observed in the near detector (Mass=1 kton, L=1km).

(c) Which beamline/detector characteristics would you change to optimise the experiment for $\nu_\mu \rightarrow \nu_\tau$ appearance?

Question 3 [6 marks]

Describe the neutrino factory concept:

(a) What are the main differences and advantages compared to a conventional neutrino beam?

(b) How can you look for CP violation effects with neutrino oscillations at a neutrino factory? Comment on the type of detector required for an appearance experiment which can measure the MNS phase δ.

(c) What are the main challenges?

Section 3: CP violation

Question 1

Draw the two possible diagrams for the process $B^- \to K^-\pi^0$, labelling the CKM matrix elements involved. What kind of CP violation can be observed in this case ?

[3 marks]

Question 2

The amplitude for the spectator (S) and dominant penguin (P) process for the $B^- \to K^-\pi^0$ decay can be expressed as:

$$\begin{aligned} S &= a\ e^{i\alpha}\ V_{ub}V_{us}^* \\ P &= b\ e^{i\beta}\ V_{tb}V_{ts}^* \end{aligned} \tag{1}$$

Given that a, b and the strong phases α and β are not affected by CP violation, show that in the Wolfenstein parameterization the CP violating asymmetry

$$A_{CP} = \frac{\Gamma(i \to f) - \Gamma(\bar{i} \to \bar{f})}{\Gamma(i \to f) + \Gamma(\bar{i} \to \bar{f})} \tag{2}$$

is proportional to $\eta\ \sin(\alpha - \beta)$. What are the conditions for this process to exhibit a significant asymmetry?

[7 marks]

Section 4: Hadron Colliders

Question 1

(a) Consider a parton-parton annihilation process producing an object of mass M, at rapidity Y at a hadron collider whose beams have the same energy operating at a CM energy of $\sqrt{s}$. What are the x values i.e the fraction of the hadron's momentum carried by the annihilating quarks expressed in terms of M, s and Y. Give 2 reasons why rapidity is a useful concept at hadron colliders. **[2 marks]**

(b) Draw a Feynman diagram for the production followed by its dominant decay for a 170 GeV mass Higgs boson at the LHC. **[2 marks]**

(c) Why is the fully hadronic final state not useful for a Higgs discovery? What final states can be used for a Higgs discovery ? List the main kinematic variables you would use and approximate cut values to select the signal **[3 marks]**

(d) Draw a Feynman diagram for a background process which has the same final state as the signal. **[2 marks]**

(e) Why might this background also be important when searching for R-parity conserving supersymmetry at the LHC? **[2 marks]**

(f) Give two reasons why R-parity is generally believed to be conserved if supersymmetry is present at TeV mass scales. **[1 mark]**

(g) Show that the invariant mass, m^2_{inv}, of a pair of particles, A and B can be written as

$$m^2_{\text{inv}} = m^2_A + m^2_B + 2\left[E^A_T E^B_T \cosh \Delta\eta - \mathbf{p}^A_T \cdot \mathbf{p}^B_T\right]$$

where $E_T \equiv \sqrt{E^2 - p^2_z}$, $\mathbf{p}_T$ is the transverse momentum 2-vector, and $\Delta\eta$ is the difference in rapidity between A and B. You may assume that $E_T \cosh \eta = E$ and $E_T \sinh \eta = p_z$. **[2 marks]**

Hence show that for massless particles with small differences in azimuthal angle, ϕ and rapidity, η, that

$$m^2_{\text{inv}} \approx p^A_T \, p^B_T \left(\Delta\eta^2 + \Delta\phi^2\right)$$

where $p_T = |\mathbf{p}_T| = \sqrt{p^2_x + p^2_y}$. **[2 marks]**

Note: $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

Section 5: QCD phenomenology

Question 1

(a) Briefly outline two alternative algorithms for defining jets, commenting on the strengths and weaknesses of each. **[8 marks]**

(b) Consider $p\bar{p}$ annihilation to produce W^+ and W^- bosons at the Tevatron. In each case at leading order the differential cross-section $d\sigma(W)/dy$, is proportional to the product of a parton distribution in the proton at momentum fraction x_1 and a parton distribution in the antiproton at momentum fraction x_2, where $x_{1,2} = x_0 \exp(\pm y)$, and y is the rapidity of the boson (see Section 4, Question1 (a)). Show that to a good approximation (explaining your approximation) that

$$A_W(y) \equiv \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy} = \frac{u(x_1)d(x_2) - u(x_1)d(x_2)}{u(x_1)d(x_2) + u(x_1)d(x_2)}. \tag{3}$$

[7 marks]

Show that for small rapidities, if $R_{du}(x) = d(x)/u(x)$, then

$$A_W(y) \approx -\frac{M_W}{\sqrt{s}} y \frac{(dR_{du}(x)/dx)_{x=M_W/sqrts}}{R_{du}(x = M_W/sqrts)}. \tag{4}$$

[5 marks]

Section 6: ILC physics

Question 1

Either from the photon propagator or a general dimensions argument, show the point-like lepton scattering cross section dependence on centre of mass energy. **[3 marks]**

Question 2

For the simple processes

$$e^+e^- \rightarrow q\overline{q} \tag{5}$$

How much more luminosity is required to observe the same number of scattering events given a linear collider operating at 1 TeV centre of mass energy, compared with a 200 GeV LEP. **[2 marks]**

Question 3

Given event samples of 5×10^6 typically are required for a precision physics analyses and the total $q\overline{q}$ cross section at 200 GeV is $\sim$ 20pb, What accelerator luminosity is required to collect the required data sample in 1 year, given that the accelerator is operational for 50% of the time. Express your answer in $\mathrm{cm}^{-1}\mathrm{s}^{-1}$. **[4 marks]**

Question 4

Sketch what would be observed in the total cross section

$$\sigma_{e^+e^-\rightarrow q\overline{q}}(\sqrt{s}) \tag{6}$$

as a function of the centre of mass energy $\sqrt{s}$, from 70 GeV to 500 GeV, just considering the Standard Model. **[4 marks]**

How would the cross section be modified by inclusion of beyond the standard model particles which couple to quarks? For each of the following scenarios draw a Feynman diagram and sketch the cross section as a function of the centre of mass energy up to 1 TeV (comparing each with the Standard Model cross section)

(a) A heavy s-channel resonance like the Z-boson of mass 750 GeV, which couples to the fermions. **[2 marks]**

(b) A quark like object heavier than the top quark, which couples to the photon/Z-boson **[2 marks]**

(c) A small unexpected modification to the photon/Z-boson coupling to quarks at high energies. **[2 marks]**