Queen Mary, University of London
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# Intercollegiate Postgraduate Course in Elementary Particle Physics

**Examination Paper 2: The Standard Model**

Thursday, 26 January, 2006

Time allowed for the Examination: 3 hours.

Answer *ALL* of the 19 questions (60 marks total).

Books and notes may be consulted.

**1.** Explain how the E. M. interaction may be considered to arise from the requirement of local gauge invariance, basing your explanation on a wave function satisfying the Dirac equation. [2 marks]

**2.** The four momenta $(k, p)$ and $(k', p')$ describe the initial and final state kinematics of $\gamma\ e^-$ Compton scattering.

Draw the two Feynman diagrams for this process, and hence write down the first order matrix elements. [4 marks]

**3.** The cross section for the Compton scattering of a virtual photon with four momentum $k = q$ $(q^2 = -Q^2)$, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}} \left[ -\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}} + \frac{2\hat{t}Q^2}{\hat{s}\hat{u}} \right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $\gamma\ e^-$ Compton scattering and $\alpha$ is the fine structure constant.

What is the origin of the $Q^2$ dependent term? [1 mark]

Modify the above so that the cross section is that for the QCD Compton process. [4 marks]

**4.** Define, in terms of the usual four vectors, the scaling variable $x$ for deep inelastic lepton - nucleon scattering (DIS) and give a simple interpretation in terms of the parton model. [2 marks]

By considering the photon-quark CMS frame explain briefly how scaling violation arises and indicate the basis for the DGLAP evolution equations. [4 marks]

**5.** Obtain an expression for the pseudo rapidity variable $\eta$ in terms of the polar angle and describe the $k_t$ clustering algorithm for jet finding. [3 marks]

**6.** How is the charged weak current expressed in the form

$$J^{+\mu} = \frac{G_F}{\sqrt{2}} \overline{u_\nu} \gamma^\mu (1 - \gamma^5) u_e$$

found to be modified in neutron or nuclear $\beta$ decay? [1 mark]

**7.** Draw Feynman diagrams for double $\beta$ decay and neutrinoless double $\beta$ decay. [1 mark]

What are the necessary conditions for neutrinoless double $\beta$ decay to occur? [1 mark]

**8.** Describe the 'seasaw' mechanism and obtain the mass eigenvalues. [3 marks]

**9.** Show that electron neutrinos ($\nu_e$) experience an additional effective potential $V_{eff}$ in matter given by

$$V_{eff} = \sqrt{2} G_F N_e$$

where $N_e$ is the electron density in matter. (You can use without proof the Fierz transformation.) [3 marks]

**10.** If the propagation of the neutrino mass eigenstates of energies $E_1$ and $E_2$ where $E^2 = p^2 + m^2$ and $m_1, m_2 << p$ is determined by a Schroedinger like equation show that the propagation in vacuo is given by

$$i\frac{d}{dt}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \left[\begin{pmatrix} p & 0 \\ 0 & p \end{pmatrix} + \begin{pmatrix} m_1/2p^2 & 0 \\ 0 & m_2/2p^2 \end{pmatrix}\right]\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

[2 marks]

**11.** If

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix},$$

obtain an expression for

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

in matter in the same form as that in vacuo. Do not attempt to multiply out your expression. [3 marks]

**12.** A spin one (vector) massless field $A^\mu$ coupling to a current $J^\mu$ satisfies the partial differential equation

$$\frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu - \partial^\mu(\partial_\nu A^{\nu)} = J^\mu$$

Modify the above to apply for a massive, virtual, spin one particle of mass $M$ interacting with the current $J^\mu$. [1 mark]

Obtain the propagator from the modified equation. [2 marks]

**13.** Show how the propagator of massive but unstable vector bosons may be modified to take account of the decay. [2 marks]

Write down the matrix element for $W^+$ decay. [1 mark]

**14.** Explain the structure of the $SU(2) \times U(1)$ electroweak theory in the massless limit, explaining as you do so, the concepts of weak isospin, weak hypercharge and local gauge invariance. [4 marks]

**15.** Write down the transformation of the $W_i^\mu$ field under local gauge transformation and the form of the self energy term. [2 marks]

Do the same for the gluon field. [2 marks]

**16.** Demonstrate that by means of a suitable symmetry breaking of a local $U(1)$ gauge symmetry of a complex scalar field, which describes charged scalar particles, that the EM field $A^\mu$ acquires mass. [3 marks]

**17.** Write down Weinberg's choice of the scalar field in $SU(2) \times U(1)$ electroweak theory before and after symmetry breaking. [1 mark]

**18.** Obtain the couplings of the Higgs to the $W$ boson [4 marks]

**19.** Explain how fermion masses may be generated in the Weinberg-Salam model and how the Kobayashi - Maskawa matrix may be related to the quark mass matrices? [4 marks]