Queen Mary, University of London
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Intercollegiate Postgraduate Course in Elementary Particle Physics

Examination Paper 2: The Standard Model

Friday, 30 January, 2004.

Time allowed for the Examination: 3 hours.

Answer *ALL* of the 20 questions (60 marks total).

Books and notes may be consulted.

1. Explain how the E. M. interaction may be considered to arise from the requirement of local gauge invariance, basing your explanation on a wave function satisfying the Dirac equation. [3 marks]

2. The four momenta (k, p) and (k', p') describe the initial and final state kinematics of $e^- \mu^-$ scattering.

Draw the Feynman diagram for this process, and hence write down the first order matrix element. [3 marks]

3. The cross section for electron - muon scattering, in the limit that masses tend to zero, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}} \left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $e^- \ \mu^-$ scattering and α is the fine structure constant. From the above obtain the differential cross section for e^+e^- to $\mu^+\mu^-$ annihilation and show that the angular distribution is of the form $(1 + cos^2\theta)$.

[4 marks]

4. Define, in terms of the usual four vectors, the scaling variables x and y for deep inelastic lepton - nucleon scattering (DIS). Give simple interpretations of the variables x and y in terms of the parton model and the lepton parton CM frame. [4 marks]

5. From the result in part 3, obtain the quark parton prediction for the cross section of the Drell Yan process, explaining the quantities you introduce. [4 marks]

6. Starting from the charged weak current expressed in the form

$$J^{+\mu} = \frac{g_w}{\sqrt{2}} \overline{u_\nu} \gamma^\mu \frac{1}{2}(1 - \gamma^5) u_e$$

show that the weak charged current may be written as

$$J^{+\mu} = \frac{g_w}{\sqrt{2}} \overline{\lambda_l} \gamma^\mu \tau^+ \lambda_l$$

where λ_l are chiral, weak isospin doublets, and τ^+ is the weak isospin raising operator.

[2 marks]

7. Show that g_w is related to the Fermi constant G_F by

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_w^2} \ .$$

[2 marks]

8. A spin one (vector) massless field A^μ coupling to a current J^μ satisfies the partial differential equation

$$\frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$$

Modify the above to apply for a massive spin one particle of mass M and show that the Lorentz condition $\partial_\nu A^\nu = 0$ is deduced. [2 marks]

9. For a massive spin one particle of mass M, energy p_0 which propagates along the z axis with momentum p_z show that

$$\begin{aligned} \epsilon_1^\mu &= (0, 1, 0, 0) \\ \epsilon_2^\mu &= (0, 0, 1, 0) \\ \epsilon_3^\mu &= \frac{1}{M}(p_z, 0, 0, p_0) \end{aligned}$$

is a suitable set of polarization vectors. [2 marks]

10. Hence or otherwise, obtain the sum over the polarization states,

$$\sum_{\lambda=1,2,3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}.$$

[2 marks]

11. Write down the propagator for a massive spin one particle and show how this may be modified for an unstable spin one particle whose full width Γ is much less than the mass M. [4 marks]

12. From the Feynman diagram write down the matrix element for W^+ decay to $e^+\nu$. [2 marks]

13. Explain the structure of the $SU(2) \times U(1)$ electroweak theory in the massless limit, explaining as you do so, the concepts of weak isospin, weak hypercharge and local gauge invariance. [4 marks]

14. Write down the covariant derivative D^μ for the above interaction and obtain the transformation for the W_i^μ field under local gauge transformation. (You may assume without proof that $D^\mu\psi$ transforms in the same way as ψ itself.) [4 marks]

15. Explain how the tri and quadrilinear W couplings arise. [2 marks]

16. Demonstrate that by means of a suitable symmetry breaking of a local $U(1)$ gauge symmetry of a complex scalar field, which describes charged scalar particles, that the EM field A^μ acquires mass. [4 marks]

17. Write down Weinberg's choice of the scalar field in $SU(2) \times U(1)$ electroweak theory before and after symmetry breaking. [2 marks]

18. Obtain the relation $M_Z = M_W/cos\theta_W$. [4 marks]

19. Explain why an electron mass term cannot occur in the $SU(2) \times U(1)$ Lagrangian and explain how fermion masses may be generated in the Weinberg-Salam model. [2 marks]

20. How is the Kobayashi - Maskawa matrix related to the quark mass matrices? [4 marks]