Queen Mary, University of London
Royal Holloway, University of London
University College London
Brunel University

Intercollegiate Postgraduate Course in Elementary Particle Physics

Examination Paper 1:

Section A: Symmetries and Conservation Laws (40 marks)

Section B: Particle Detectors (20 marks)

Tuesday, 28 January, 2003.

Time allowed for the Examination: 3 hours.

Answer *ALL* questions. Books and notes may be consulted.

Section A: Symmetries and Conservation Laws

Question 1 [20 marks]

(a) Justify the application of SU(3) in describing hadron states made from u, d and s quarks. Explain the approximations involved. [1 mark]

(b) Consider the SU(2) description of qq states made from u and d quarks. Use Young Tableaux to identify the states of well-defined symmetry - use the Tableaux to identify the multiplicities, show how they can be used to identify the states (by assigning labels to the boxes) and give the corresponding flavour wave-functions. [2 marks]

(c) Explain how one goes from the representation for an SU(2) flavour doublet

$$\mathbf{2} = \begin{pmatrix} u \\ d \end{pmatrix}$$

to the conjugate representation $\bar{\mathbf{2}}$. In particular, give the form of the vector and explain (without providing a detailed justification) why it has this form.
In analogy with part (b), identify the $q\bar{q}$ states. [3 marks]

(d) Now consider SU(3) of flavour (u,d,s) for qq states. Again, use Young Tableaux to identify the states of well-defined symmetry. [3 marks]

(e) Finally, consider SU(3) of flavour (u,d,s) for $q\bar{q}$ states. Use Young Tableaux to identify the multiplets and their multiplicity. Write down suitable wave-functions. (However, in this case, do not attempt to use the Young Tableaux to describe the symmetries of the states.) [3 marks]

(f) The singlet SU(3) flavour state for three quarks is

$$uds + dsu + sud - sdu - usd - dus$$

Demonstrate that this is invariant under SU(3) transformations.
It will suffice to consider infinitesimal SU(3) transformations

$$U = \exp\left(i\epsilon \cdot \lambda\right) \approx 1 + i\epsilon \cdot \lambda$$

(*Hint:* Consider the transformations generated by λ_1, λ_2 and λ_8 separately along with a suitable use of "likewise". Under a transformation U, $q_1 q_2 q_3 \to U(q_1)U(q_2)U(q_3)$ - where the operator acts separately on the three individual quark states. The λ matrices are provided on the following page.) [6 marks]

(g) Show how the multiplicities of the three-quark SU(3) flavour states are obtained from Young Tableaux by combining three single-quark representations. Explain briefly the correspondence with the observed baryon states.
The singlet considered in part (f) is not observed in the flavour representation; in what form is the SU(3) singlet observed ? [2 marks]

The fundamental quark representations are:

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and the SU(3) generators are:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Question 2 [20 marks]

(a) Draw the unitarity triangle, labeling its sides and angles. Describe briefly how the CKM elements V_{ub} and V_{cb} can be measured. [3 marks]

(b) Explain how the presence of neutral B mixing can give rise to indirect CP violation. Give an example of a decay channel which exhibits this effect. [4 marks]

(c) Draw Feynman diagrams for the tree and penguin processes via which the decay $B^0 \to \mathrm{J}/\psi\pi^0$ can occur, labelling the CKM matrix elements. What kind of CP violation may be observed in this decay? [5 marks]

(d) Consider a decay $i \to f$ which has contributions from two amplitudes. Each amplitude has the form:

$$A_i = |A_i| \exp(i\phi_i) \exp(i\Delta_i)$$

where the phases ϕ_i come from weak interaction effects (the CKM matrix) and the phases Δ_i come from strong interaction effects.

(i) State how each of the contributions to the phase for the charge conjugate decay ($\overline{\phi}_i$ and $\overline{\Delta}_i$) are related to ϕ_i and Δ_i within the standard model.

(ii) Obtain an expression for the CP violating asymmetry:

$$A_{CP} = \frac{\Gamma(i \to f) - \Gamma(\overline{i} \to \overline{f})}{\Gamma(i \to f) + \Gamma(\overline{i} \to \overline{f})}$$

(iii) Determine the conditions for CP violation to be observed.

[8 marks]

Section B: Particle Detectors

Question 1 [10 marks]

The figure shows the detector ALEPH which operated at the LEP electron- positron collider during the 1990's. By considering the decays $Z \to e^+e^-$ and $Z \to$ hadrons, in the region close to a rapidity of zero, explain how these events were reconstructed using the different detector systems.

In your answer you should discuss the advantages and disadvantages of the techniques that were used to construct the following sub-systems: the vertex detector, the electromagnetic calorimeter, the hadron calorimeter.

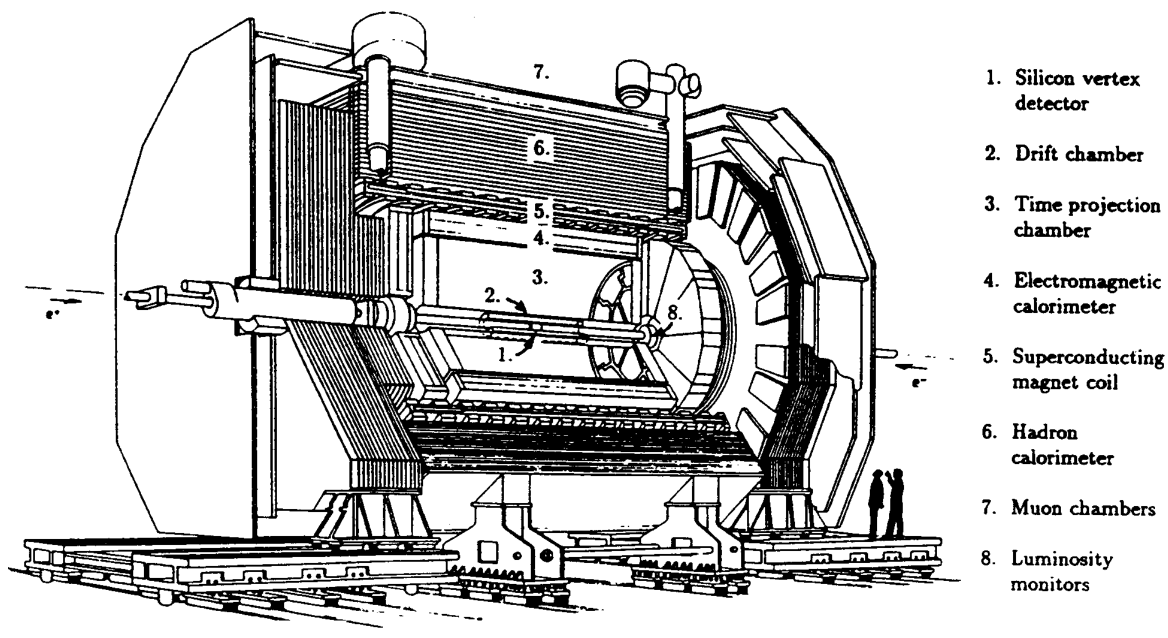

Figure 1: The ALEPH detector.

Question 2 [10 marks]

Which channels of decay of a standard model Higgs boson of 120 GeV mass or of 200 GeV mass would be expected to be detected and well measured in a few years of running

(a) at a 500 GeV linear e^+e^- collider?

(b) at the LHC?

Explain the differences between the machines and the masses in the sensitivity to the different decays. [5 marks]

In a particular new model the Higgs boson has a mass of 130 GeV and decays 90% of the time to invisible particles, having been produced in the normal Higgstrahlung process $e^+e^- \to HZ$. How could such a Higgs boson be detected. Draw the lowest order Feynmann graph for the production process. Deduce from this which of the standard Higgs decay channels must still be present, despite the nonstandard couplings of the model.

[5 marks]