Queen Mary & Westfield College
Royal Holloway UoL
University College
Brunel University

Intercollegiate Postgraduate Course in Elementary Particle Physics

Paper 1: Thursday, 20 January 2000

Time allowed for the Examination: 3 hours

Attempt THREE *questions - One from each section*

Section A: Symmetries, Conservation Laws and B Physics
Section B: Electroweak Interactions part 1
Section C: LEP 2 Physics

Question 1

(a) Explain what is meant by the term "generator" of a unitary group. **2 marks**

(b) Derive the structure constants of the $SU(2)$ group using its generators. **4 marks**

(c) In QCD it is postulated that all observable hadrons are colour singlets. How does this explain the pattern of hadrons we observe? **4 marks**

(d) Explain how the observed pattern of fundamental fermions and bosons of the Standard Model can be explained using group theory. Explain carefully any group theory terminology that you use. **6 marks**

(e) Describe, generally, how the symmetries of the Standard Model might be extended in a GUT model such as $SU(5)$. What would be the experimental signatures? **4 marks**

Question 2

Describe how the study of the B mesons allows us to measure several fundamental parameters in the Standard Model. Your answer should be illustrated with examples of specific measurements that have been made or have been proposed for existing or future experiments, together with a description of the experimental difficulties they present.

Question 3

(a) By demanding that solutions of the Dirac equation be invariant under the local space - time transformation $\psi^{'}(x,t) = exp(iq\chi(x,t))\psi(x,t)$ show that a 4-vector field A^μ, satisfying the gauge transformation $A^{\mu^{'}} = A^\mu - \partial^\mu\chi$ must be introduced, where $\chi(x,t)$ is a scalar function.

(b) The four momenta (k,p) and $(k^{'},p^{'})$ describe the initial and final state kinematics of $e^-\mu^-$ scattering.

Write down the first order matrix element for this process.

Show that the cross section for electron - muon scattering, in the limit that masses tend to zero, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $e^-\ \mu^-$ scattering and α is the fine structure constant.

(c) Show that annihilation cross section for e^+e^- to $\mu^+\mu^-$ is given by

$$\sigma = \frac{4\pi\alpha^2}{3s}.$$

(d) Describe as far as you can, how Gauge Invariance determines the colour interactions of quarks and gluons. Draw the Feynman diagrams for these interactions.

Question 4

(a) Draw Feynman diagrams and write down the matrix elements for the following charged current weak processes, assuming a pointlike weak interaction.

$\nu_\mu + d \to u + \mu^-$,

$\nu_\mu + d \to c + \mu^-$ and

$\pi^+ \to \mu^+ + \nu_\mu$.

(b) Determine an expression for the decay rate of the π^+, and obtain an expression for the ratio of the K^+ and π^+ decay rates.

(c) Show that a massive, free, spin one particle field satisfies the Lorentz condition. If a massive spin one particle of mass M and energy p_0 is propagating along the z axis with momentum p_z show that

$$\begin{aligned} \epsilon_1^\mu &= (0,1,0,0) \\ \epsilon_2^\mu &= (0,0,1,0) \\ \epsilon_3^\mu &= \frac{1}{M}(p_z,0,0,p_0) \end{aligned}$$

is a suitable set of polarization vectors.

Hence obtain the sum over the polarization states,

$$\sum_{\lambda=1,2,3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}.$$

(d) Write down the matrix element for W^+ decay to $e^+\nu$ and evaluate the "matrix element squared" as far as the trace calculation.

(e) Draw the Feynman graphs for the production of W bosons in $p-p$ and $p-\overline{p}$ colliders and explain why the W bosons are polarized at production.

Section C : LEP 2 Physics

Question 5

(a) Draw the three Feynman diagrams giving rise to W-pair production at LEP2 and sketch the form of the cross section as a function of $\sqrt{s}$. Indicate qualitatively how the cross section would vary when:
(i) The mass of the W increases
(ii) The width of the W increases. [3]

(b) Draw the Feynman diagrams which contribute to the width of the W-boson and hence derive the total width of the W in terms of the partial width to the $e\bar{\nu}_e$ final state. Discuss qualitatively how you would expect this result to change if there were significant Cabibbo-like mixing between quarks of the first and third generation. [4]

(c) Discuss how, within the standard model, the W-mass depends on the masses of the top quark and the Higgs boson. Draw the relevant Feynman diagrams which give rise to this dependence and explain qualitatively why the upper and lower limits differ in the present estimates of the Higgs mass $m_H = 76^{+85}_{-47}$ GeV [3]

(d) Explain why the existence of such a light Higgs boson would be "unnatural" in the standard model, whereas no such problem exists for light fermions, nor for light electroweak gauge bosons. [4]

(e) **either**
The present lower limit on the mass of the Higgs boson is obtained from direct experimental searches at LEP2 and presently stands at approximately 108 GeV. Give an account of what these searches entail. Include a brief discussion of why silicon vertex detectors are so important in these searches and discuss the most significant backgrounds.
or
What is meant by *supersymmetry*? Explain qualitatively how supersymmetry helps to keep the Higgs boson light and discuss the general experimental signatures for supersymmetry at e^+e^- colliders. [6]