Queen Mary & Westfield College
Royal Holloway UoL
University College

Intercollegiate Postgraduate Course in Elementary Particle Physics

Paper 2: Thursday, 25 March 1999

Time allowed for the Examination: 3 hours

Attempt THREE *questions - One from each section*

Section A: Electroweak Interactions part 2
Section B: Statistical Analysis of Data
Section C: Experimental Techniques

Section A : Electroweak Interactions part 2

Question 1

(a) Starting from the charged weak current expressed in the form

$$J^{+\mu} = \frac{g_w}{\sqrt{2}} \overline{u_\nu} \gamma^\mu \frac{1}{2}(1-\gamma^5) u_e$$

show that the weak charged current may be written as

$$J^{+\mu} = \frac{g_w}{\sqrt{2}} \overline{\lambda_l} \gamma^\mu \tau^+ \lambda_l$$

where λ_l are lepton weak isospin doublets, and τ^+ is the weak isospin raising operator.

Show that g_w is related to the Fermi constant G_F by

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_w^2}$$

(b) Explain what is meant by weak hypercharge Y where $Y = 2Q - 2T^3$.

(c) Starting from the interaction term

$$g_w J^{\mu 3} W_\mu^3 + \frac{g^{'}}{2} J_Y^\mu B_\mu$$

and making the massless W^3 and B fields superpositions of the photon and Z^0 fields show that the neutral current coupling is given by

$$\frac{g_w}{cos\theta_w} \overline{u_f} \gamma^\mu \frac{1}{2}(c_V^f - c_A^f \gamma^5) u_f$$

where $c_V^f = T_f^3 - 2sin^2\theta_w$ and $c_A^f = T_f^3$. In the above θ_w is the Weinberg angle, T^3 is the third component of the weak isopin and the suffix f signifies a fermion.

(d) Write down the propagator for a massive spin one particle and show how this may be modified for an unstable spin one particle whose full width Γ is much less than the mass M.

(e) Write down the matrix element for e^+e^- annihilation through the Z^0 to $f\overline{f}$ fermion pairs. Evaluate the "matrix element squared" ,summed over the spin states and averaged over the initial spins, as far as the trace evaluation.

(f) In a type 2 supernova explosion the iron core at the centre of a star collapses into a neutron star.

What is the reaction that converts the protons in the iron core into neutrons?

The gravitational energy released by the collapse heats the neutron star and the radiation field photons materialize into e^+e^- pairs. What is the mechanism that converts most of the gravitational energy released into neutrinos?

State the kind of neutrinos emitted, and suggest the reactions by which such neutrinos may be detected in a large water Cerenkov detector.

Question 2

(a) Define, in terms of the usual four vectors, the scaling variables x and y for deep inelastic lepton - nucleon scattering (DIS). Give simple interpretations of the variables x and y in terms of the parton model and the lepton parton CM frame.

(b) Explain what is meant by scaling in deep inelastic processes.

(c) For Q^2 much smaller than M_W^2 the neutrino deep inelastic cross section is given by

$$\frac{d^2\sigma^{\nu\overline{\nu}}}{dxdy} = \frac{G_F^2}{2\pi}s\left[F_2\frac{(1+(1-y)^2)}{2} \pm xF_3\frac{(1-(1-y)^2)}{2}\right]$$

where s is the square of the CMS energy, F_2 and F_3 are the structure functions, and G_F is the Fermi constant. (In this question the Cabibbo angle is ignored).

If the ν quark and ν antiquark cross sections are given by

$$\frac{d\sigma^\nu}{d\Omega} = \frac{G_F^2}{4\pi^2}\hat{s} \text{ and } \frac{d\sigma^\nu}{d\Omega} = \frac{G_F^2}{4\pi^2}\hat{s}(1+cos\theta)^2$$

where $\hat{s}$ and θ are respectively the square of the CM energy and the scattering angle in the lepton quark CM frame, show that

$$F_2^\nu = 2x(Q(x) + \overline{Q(x)}) \text{ and } xF_3^\nu = 2x(Q(x) - \overline{Q(x)})$$

here $Q(x)$ and $\overline{Q(x)}$ are the appropriate quark probability distributions within the nucleon.

(d) At HERA 27.6 GeV electrons are in head on collision with 920 GeV protons. Describe the appearence of events in the detector for $x \ll 0.03$, $x = 0.03$ and $x \gg 0.03$.

(e) Draw and label the Feynman diagrams of two processes that lead to scaling violations.

(f) Write down the matrix element for Compton scattering and then write down the corresponding matrix element for QCD Compton scattering.

Section B : Statistical Analysis of Data

Question 3

For this question, answer all of (a)–(d) and either (e) or (f).

Consider N independent Poisson variables $n_1, \ldots, n_N$ with mean values $\nu_1, \ldots, \nu_N$. Suppose the mean values are related to a controlled variable x by

$$\nu(x) = \theta x,$$

where θ is an unknown parameter. The values of ν_i for $i = 1, \ldots, N$ are thus given by $\nu_i = \nu(x_i) = \theta x_i$, where the values $x_1, \ldots, x_N$ are known. We are given a data sample consisting of $n_1, \ldots, n_N$. (An example of this situation is where we measure the numbers of neutrino scattering events produced at N different values of the neutrino energy. The variable x is proportional to the energy and θx is the predicted number of events.)

Answer all of (a)–(d):

(a) (4 points) Show that the log-likelihood function is

$$\log L(\theta) = \sum_{i=1}^{N} (n_i \log \nu_i - \nu_i)$$

and hence that the maximum-likelihood estimator for θ is given by

$$\hat{\theta} = \frac{\sum_{i=1}^{N} n_i}{\sum_{i=1}^{N} x_i}.$$

(b) (3 points) Show that $\hat{\theta}$ is an unbiased estimator for θ.

(c) (4 points) Find the variance of $\hat{\theta}$ and show that it is equal to the RCF bound.

(d) (4 points) Use the method of least squares to find an estimator for θ. For the denominators in the χ^2 use the Poisson variances $\sigma_i^2 = \nu_i$.

Answer one of (e) or (f) (5 points):

(e) Explain how to evaluate the goodness-of-fit for both the maximum-likelihood and least-squares cases. Comment on what must be done if some of the n_i are small or zero.

(f) Suppose n_i represents the number of events found with neutrino beam energy E_i and integrated luminosity L_i. The expected number of events can be written as

$$\nu_i = \theta E_i L_i \, .$$

Suppose data can be taken at two different energies, E_1 and E_2, and the total luminosity $L = L_1 + L_2$ is fixed by budget constraints. How should the luminosities L_1 and L_2 be shared between the two energies so as to minimize the error in the estimate of θ?

Section C : Experimental Techniques

Question 4

Summarise the contributions of *precision silicon tracking devices* in experimental particle physics over the last decade. Illustrate your answer with examples (a) of detector details from specific experiments, and (b) of physics channels and areas of study that are particularly dependent on such instrumentation. Comment on the proposed future role of silicon tracking in LHC experiments, and explain the features they must have to allow them to operate satisfactorily over long periods at such colliders.

Explain how silicon microstrip detectors work, with reference to signal production, substrate properties and tracking precision. Briefly describe the substrate processing sequences that are required to manufacture the most simple devices which are capable of being readout with a/c coupling. Discuss the differences between devices with single-sided and double-sided readout structures.

Derive the *semiconductor equation* that relates the densities of electron and hole densities, $\mathrm{np} = \mathrm{n}_i^2$, and explain the meaning of *depletion layer* when applied to a biassed device.

Question 5

Sketch the structure of a typical (or actual) detector at a colliding beam machine including the sub-detectors.

Explain the role of each sub-detector with particular emphasis on its relative position within the detector.

A typical LEP detector comprises a barrel and two endcaps with overall linear dimensions of about 10m. Justify this structure, including the dimensions. Explain why LHC detectors are about three times larger in linear dimensions.

Discuss the relative advantages and disadvantages of crystal and sampling electromagnetic calorimeters.

Question 6

What are the problems of triggering and data acquisition for experiments at the CERN Large Hadron Collider and how would you address them? Answer the question with reference to a block diagram showing how data is communicated and processed at multiple levels. Discuss likely technologies that may be used.