Queen Mary & Westfield College
Royal Holloway UoL
University College

# Intercollegiate Postgraduate Course in Elementary Particle Physics

Paper 1: Monday, 17 March 1997

*Attempt* THREE *questions - One from each section*

Time allowed for the Examination: 3 hours

**Section A:** Symmetries and Conservation Laws
**Section B:** Electroweak Interactions
**Section C:** Computing in High Energy Physics

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Dirac spinors:

$$(\not p - m)u(p) = 0, \qquad \sum_s u(p,s)\bar{u}(p,s) = \not p + m$$

$\gamma$ matrices:

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}, \qquad \gamma^0\gamma^{\mu\,\dagger}\gamma^0 = \gamma^\mu$$

Definition of $\gamma^5$ and its basic properties:

$$\begin{aligned} \gamma^5 &= i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \gamma^{5\,\dagger} = \gamma^5, \qquad \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0, \\ P_L &= \frac{1}{2}(1-\gamma^5), \quad P_R = \frac{1}{2}(1+\gamma^5), \quad P_L + P_R = 1, \qquad P_L^2 = P_R^2 = 1, \quad P_L P_R = 0 \end{aligned}$$

Trace theorems:

$$\begin{aligned} \mathrm{Tr}\,1 &= 4, \quad \mathrm{Tr}\,\gamma^\mu\gamma^\nu = 4g^{\mu\nu}, \quad \mathrm{Tr}\,\gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\rho = 4\left(g^{\lambda\mu}g^{\nu\rho} + g^{\lambda\rho}g^{\mu\nu} - g^{\lambda\nu}g^{\mu\rho}\right), \\ \mathrm{Tr}\,\gamma^5 &= 0, \quad \mathrm{Tr}\,\gamma^5\gamma^\mu = 0, \quad \mathrm{Tr}\,\gamma^5\gamma^\mu\gamma^\nu = 0, \quad \mathrm{Tr}\,\gamma^5 \not a \not b \not c \not d = i\varepsilon_{\lambda\mu\nu\rho}a^\lambda b^\mu c^\nu d^\rho \end{aligned}$$

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## Question 1.

(i) State the four group axioms. (2 marks)

(ii) Explain what is meant by the term "generator" of a unitary group. (2 marks)

(iii) Explain what is meant by the "Lie algebra" of a group. What are the structure constants? (2 marks)

(iv) What is meant by the "fundamental" and "adjoint" representations of a Lie group? Illustrate your answer by using the weak interactions of quarks and leptons. (4 marks)

(v) The strong interaction has an approximate symmetry in which the light quarks $u, d, s$ form a triplet corresponding to the fundamental representation of a $SU(3)$ flavour group. Using Young's Tableaux, or otherwise, find the products of representations corresponding to the mesons and baryons *i.e.* combinations of $q\bar{q}$ and $qqq$. How do these multiplets correspond to the observed physical states? Draw a diagram of meson multiplets, with the physical states labelled. Explain the consequences of also considering the quark spin and the $SU(3)$ colour symmetries. (10 marks)

## Question 2.

(i) Explain how the magnitudes of the CKM matrix elements $V_{cb}$ and $V_{ub}$ have been measured. (4 marks)

(ii) Write down the CKM matrix in the Wolfenstein parameterization. (2 marks)

(iii) Draw the Unitary Triangle, labelling the sides and angles. What is the area of the triangle in terms of the Wolfenstein parameters? (2 marks)

(iv) Draw the diagrams for a B meson decay in which we might expect to observe $CP$ violation due to the interference of a spectator with a penguin process. Indicate on your diagrams the CKM matrix elements involved. Explain why such processes are difficult to relate directly to the Unitary Triangle. (5 marks)

(v) Describe how the presence of neutral B meson mixing gives rise to the possibility of indirect $CP$ violation that may be directly related to the CKM matrix. Give examples of decay channels that can be used to measure the three angles of the Unitary Triangle you have drawn for part (iii). How, experimentally, would one look for a $CP$ asymmetry? What are the experimental challenges of such measurements? (7 marks)

## **Section B:** Electroweak Interactions

### Question 3.

(i) Draw the Feynman diagrams in lowest order of perturbation theory for the following processes:
a) $e^+ + e^- \to \gamma\gamma$; $e^+ + e^- \to \gamma\gamma\gamma$
b) $e^+ + e^- \to e^+ + e^-$; $e^+ + e^- \to \mu^+ + \mu^-$
c) $e^+ + e^- \to f + \bar{f}$ where $f$ is any fermion.
d) $\nu_\mu + e^- \to \nu_e + \mu^-$
Label all lines, including internal lines, with the corresponding particle labels.

(ii) Describe the procedure of spin summation in the evaluation of the modulus squared of the scattering amplitude. Explain which particle spins are summed over and which are averaged over.
Comment on the statement:

> *"The traces of products of Dirac $\gamma$ matrices, which appear in the evaluation of the modulus squared of the scattering matrix, arise as a result of spin summation".*

(iii) Evaluate the spin-averaged mod-squared of the amplitude for pion decay, $\pi^- \to \mu^- + \bar{\nu}_\mu$, which is given by

$$\mathcal{M} = \frac{G}{\sqrt{2}} f_\pi \left(p^\mu + k^\mu\right) \left[\bar{u}(p)\gamma_\mu(1-\gamma_5)v(k)\right]$$

where $G$ is the Fermi constant, $f_\pi$ is the pion decay constant, $p$ and $k$ are the 4-momenta of the muon and antineutrino, respectively, and all other symbols are in standard notation.
You should use the Dirac equation, trace theorems and other "useful formulæ" from the rubric.

### Question 4.

(i) State the basic assumptions of the $(V-A)$ theory of the weak interactions. Write down the weak lepton current in terms of the Dirac spinors appropriate for this theory and draw the corresponding Feynman graph for $\nu_e e^-$ elastic scattering, carefully labeling all the elements of the Feynman graph.

(ii) Explain how the absence of helicity $+1/2$ states of the neutrino (and the absence of helicity $-1/2$ states of the antineutrino) leads directly from the Fermi theory of the weak interactions to the $(V-A)$ theory. Using the properties of the $\gamma^5$ matrix given in the rubric, show that only left-handed electrons couple to neutrinos.

(iii) Discuss the need for a neutral intermediate vector boson in a theory that unifies the electromagnetic and weak interactions.

(iv) Describe an experiment to establish the existence of weak neutral current interactions.

**Question 5.**

The probability distributions for (a) the Gaussian, (b) the binomial and (c) the Poisson distributions are

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\overline{x})^2}{2\sigma^2}\right), \quad P(r) = \frac{N!}{r!(N-r)!} p^r (1-p)^{N-r}, \quad P(r) = \frac{\lambda^r}{r!} \mathrm{e}^{-\lambda}$$

respectively.

Explain the typically circumstances in which each of these can be used, stating the meaning of the symbols in each distribution.

In what circumstances can the binomial and Poisson distributions be approximated by a Gaussian, and what are the corresponding values for $\overline{x}$ and $\sigma$ in those circumstances?

Show that for the Poisson distribution the expectation value of $r$ is given by

$$< r > \equiv \sum_{r=0}^{\infty} r P(r) = \lambda.$$

Define the quantities chi-squared ($\chi^2$) and number of degrees of freedom ($\nu$) that appear when a comparison is made between a set of data points and a model.

Make a rough sketch of the probability density function $f(\chi^2, \nu)$ for $\nu = 1$, 4 and 10.

Define the quantity "chi-squared probability" and explain why it is normally quoted in preference to chi-squared.