Worked Example
Heat Source near an Insulated Wall

Hold a heat source of strength Q at $\mathbf{x}_0 = (x_0, y_0, z_0)$ near an *insulated* plane wall, i.e., one through which no heat can pass, at $x = 0$. We must then have no component of heat flux through the wall; i.e., $\mathbf{n}\,\boldsymbol{.}\,(-k\nabla T) = 0$ on the wall. Therefore we must solve

$$\nabla^2 T = -\frac{Q}{k}\delta(\mathbf{x} - \mathbf{x}_0) \qquad \text{in } x > 0$$

subject to

$$\frac{\partial T}{\partial n} = 0 \qquad \text{on } x = 0.$$

This is a problem with Neumann (rather than Dirichlet) boundary conditions.

We use the method of images. Introduce an image source of strength $+Q$ at $\mathbf{x}_1 = (-x_0, y_0, z_0)$. (Note that for Dirichlet boundary conditions we would have used $-Q$ for the strength of the image.) Because ∇T is radial from each source, the total ∇T (from the two sources combined) must have zero component perpendicular to the wall. Hence we have $\partial T/\partial n = 0$ as required. Therefore (by uniqueness) the solution is

$$T = \frac{Q}{4\pi k}\left\{\frac{1}{|\mathbf{x} - \mathbf{x}_0|} + \frac{1}{|\mathbf{x} - \mathbf{x}_1|}\right\}.$$