UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M234: Electricity and Magnetism

COURSE CODE	:	**MATHM234**
UNIT VALUE	:	**0.50**
DATE	:	**28-APR-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

06-C1049-3-40

TURN OVER

All questions may be attempted but only marks obtained on the best ***four*** *solutions will count.*
The use of an electronic calculator is ***not*** *permitted in this examination.*

1. Consider the non-relativistic motion of a particle of mass m and charge q in a zero electric field $\mathbf{E} = (0, 0, 0)$ and a time-independent magnetic flux density $\mathbf{B}$.

   (a) State the equation of motion.

   (b) Prove that the kinetic energy of the particle is conserved.

   (c) Find the general solution for the particle path ($\mathbf{r} = \mathbf{r}(t)$) in the case when $\mathbf{B} = (0, 0, B_0)$, where B_0 is a constant. Describe and sketch a typical path.

2. Consider two concentric spherical shells with radii a and b, with $a < b$. Suppose that both spherical shells are uniformly charged, with total charge Q_a on the inner shell and total charge Q_b on the outer shell. Starting from the vacuum version of Maxwell's equations in the electro-static limit, determine the following:

   (a) the electric field $\mathbf{E}$ everywhere, assuming that $|\mathbf{E}|$ tends to zero at infinity;

   (b) the corresponding electric potential ϕ;

   (c) the capacitance in the case where $Q_a = -Q_b$.

3. Throughout this question, the vacuum versions of Maxwell's equations are assumed.

   (a) Determine the electrostatic energy U_e in a parallel plate capacitor of plate area A and plate separation d when the plates have equal and opposite charges of magnitude Q. State clearly any standard approximations used. Sketch the physical system.

   (b) Determine the magnetostatic energy U_m in a long thin circular cross-sectional solenoid of length ℓ and radius a with n turns per unit length, when the wire is carrying a current I. State clearly any standard approximations used. Sketch the physical system.

   (c) Assuming that the solutions for parts 3a and 3b are approximately valid for the time-dependent case, and that each end of the wire from the solenoid is connected to a different plate of the capacitor, show that this system supports a sinusoidal oscillation and determine its frequency. You may assume that energy is conserved, but any other assumptions should be clearly stated. Where might such a tuned circuit be found in your home?

4. A consequence of the vacuum equation $\operatorname{curl}\mathbf{E} = -\partial\mathbf{B}/\partial t$, is that

$$\oint_{C(t)} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{r} = -\frac{d}{dt} \int_{S(t)} \mathbf{B} \cdot \mathbf{n}\, dS,$$

where $S(t)$ is a time-dependent surface element with unit normal field $\mathbf{n}$ and closed bounding curve $C(t)$, and $\mathbf{v}$ is the velocity of a point on $C(t)$.

(a) Verify this result in the case where $\mathbf{E}$ and $\mathbf{B}$ are given in cylindrical polar coordinates (r, θ, z) by

$$\mathbf{E} = \hat{\theta} \exp(-t), \qquad \mathbf{B} = \hat{\mathbf{z}}\, r^{-1} \exp(-t),$$

and $C(t)$ is the circle $z = 0$, $r = 1 + t$, where $\hat{\theta}$ and $\hat{\mathbf{z}}$ are respectively the unit vectors in the θ and z directions.

(b) What is the interpretation of $\mathbf{E} + \mathbf{v} \times \mathbf{B}$ in the frame moving at velocity $\mathbf{v}$?

5. (a) State the electromagnetic media form of Maxwell's equations in differential form, giving the definitions of the fields $\mathbf{D}$ and $\mathbf{H}$. What are the physical interpretations of the polarization field $\mathbf{P}$ and magnetization field $\mathbf{M}$?

(b) Determine the fields $\mathbf{E}$ and $\mathbf{D}$ everywhere for a system consisting of a uniformly polarized ball of radius a with constant polarization $\mathbf{P}_0$.

6. (a) State the defining property of a homogeneous isotropic conductor of conductivity σ.

(b) Show that the magnetic flux density $\mathbf{B}$ in such a conductor evolves according to

$$\nabla^2\mathbf{B} = \mu_0\sigma\frac{\partial \mathbf{B}}{\partial t} + \varepsilon_0\mu_0\frac{\partial^2 \mathbf{B}}{\partial t^2}.$$

(c) Solve for the $\mathbf{B}$ field for an electromagnetic plane wave in such a conductor, where all fields are assumed to be proportional to $\exp(i(\mathbf{k} \cdot \mathbf{x} - wt))$. Find the lengthscale of decay of the $\mathbf{B}$ field in the direction of motion, and give the name for this lengthscale.