Diploma in Astronomy—First Term 2004-5

Solar System: Problem Paper 3–Model Answers.

23rd November 2004

Solutions should be returned by Tuesday December 7th. After that date they can be accepted, but once model solutions are posted, they cannot receive credit for marks (but can count towards completion of the course). The weighting for each part of each question is given in ().

1. Imagine you were an engineer in charge of landing an astronaut on Venus, who would have to be able to descend through the atmosphere and then move around on the surface taking samples of rocks, atmosphere, etc., and that survival for 24 hours was required, followed by ascent back to a spacecraft. List, in a little essay or paragraph, the specific hazardous conditions that exist in the atmosphere or on the surface, and outline what means the environment suit might use to provide protection against them. [200-300 words] (10)

Answer: Some latitude is allowed in the answers to this essay question, but for full marks the answer should mention the following Venusian hazards: protection against high temperature (may require thick asbestos lining and/or nuclear-powered air conditioner), protection against high pressure (may require armour similar to extreme deep-sea suit), breathable air supply and sealant against CO_2 leakage. Other hazards include protection against hot sulfuric acid droplets in the clouds during descent and ascent (i.e, in the atmosphere). There is a penalty (subjectively assessed according to lecturer-annoyance factor) for going substantially over the 300 word limit. A complete answer under the 200-word limit is not penalised.

Award up to 2 marks for each principal hazard (T, P, air) and up to 2 marks for all additional requirements (acid protection, food, water). Award up to 2 marks for lack of errors in English composition.

2. The Galilean Moons are said by experts to have a “Laplace resonance” in their periods, close to but not exactly the simple ratio of 1:2:4 for Io (satellite 1), Europa (2) and Ganymede (3) as given in the text.

(a) This part is about using the right number of significant figures in a calculation. If the mean motion n is defined as $n = 360°/\text{Period(days)}$ and $P_1 = 1.769138\,\text{d}$, $P_2 = 3.551181\,\text{d}$, and $P_3 = 7.154553\,\text{d}$, verify to an appropriately correct number of significant figures that the Laplace resonance

$$n_1 - 3n_2 + 2n_3 = 0$$

is indeed precise. (6)

(Show your working, don’t just say, “It is verified,” without demonstrating it.)

(b) By what percentage is the 1:2:4 approximate resonance not accurate? (4)

Answer: The information is given to 7 significant figures for each period, so we want our verification to be that good. We recognise that the final digit may be uncertain by ± 0.5 so we should not expect *perfect* agreement. To avoid rounding errors, ideally we should carry out the calculations to 8 significant places and round to 7 in the final result, but if you simply carried 7 significant figures that was accepted. Substitute the periods given in the equation

$$360/P_1 - 3 \times 360/P_2 + 2 \times 360/P_3 = 0$$

to obtain (carrying 8 significant digits)

$$203.48893 - 304.12418 + 100.63522 = -0.00003$$

which is 000.0000 to 7 significant figures. Thus we have verified that the Laplace resonance is precise at this level of accuracy.

(b) If the 1:2:4 relationship (given in the textbook) was exact, then for example, we would have $(P_2 - 2 \times P_1)/P_2 = 0.000000$ which should be exact. But it isn't: you get 0.00363 or an approximate error of 0.36 %.
There are two other pairs of values to check:
$(P_3 - 2 \times P_2)/P_3 = 0.00729$ or 0.73%;
$(P_3 - 4 \times P_1)/P_3 = 0.01090$ or 1.09%.

So we conclude that the approximate relationships in the textbook are in error by around 1% or a bit less.