# Diploma in Astronomy—First Term 2004-5

## Solar System: Problem Paper 1. Model Answers.

10th November 2004

Solutions had to be returned by Tuesday 9 November. After that date they can be accepted, but once these model solutions are posted, they cannot receive credit for marks (but can count towards completion of the course). The weighting for each question is given in ().

1. A (fictitious) superior planet, Kerplooie (they had a very strange language), was observed from Earth by the ancient Baboonians over many centuries. They found an average synodic period of 1187.716 days between oppositions. Assuming the sidereal period of Earth is 365.256 days (as given in lectures), calculate the sidereal period of Kerplooie. (5)

**Answer:** All the questions can be answered by reference to the lecture notes now on the web site. Let $P_K$ be the sidereal period of Kerplooie, $Y$ be the sidereal period of Earth, and $S_K$ be the synodic period of Kerplooie, then

$1/P_K = 1/Y - 1/S_K$

and following the Mars example,

$1/P_K = 1/365.256 - 1/1187.716 = 0.0018958986706$

and $P_K = 527.467$ days (Earth days, of course).

For full marks the answer must retain all the significant figures of the original data (6 figures for the least precise number), no more, no less.

2. Use the results from (1), plus Kepler's third law, $P^2 = ka^3$, to deduce the semi-major axis $a$ of Kerplooie's orbit in astronomical units. Explain your choice of units and state what value of $k$ should be adopted (see lecture notes). How does Kerplooie's $a$ compare with Mars' ($a = 1.524$ AU)? (5)

**Answer:** To use Kepler's third law, convert $P$ to sidereal years and get the result for $a$ in astronomical units (AUs). The value of $k$ in these units is 1.

$P_K = 527.467/365.256 = 1.44410$ years (retain significant figures)

$a = \sqrt[3]{1.44410^2} = 1.27761$ AU

This is significantly smaller than Mars' orbit, so we expect Kerplooie to be warmer than Mars but cooler than Earth. Here, I allow full marks for getting $a$ to 4 significant figures (1.278).

3. In the lectures on Earth we talked about the Greenhouse Effect and the way in which a planet's expected mean temperature can be calculated. Assume that Kerplooie is a "terrestrial" planet with a nearly circular orbit. Use the semi-major axis of Kerplooie's orbit from (2), assume that its rotation period is 22.5 Earth hours (what does this imply for the temperature

calculation?), assume that its Bond Albedo is 0.27, and assume that it has a Greenhouse temperature increase of 28 degrees C (same as K) due to $CO_2$ and $H_2O$ in its atmosphere.

Deduce the mean temperature of Kerplooie.

Hint: Remember that the Solar Constant for Earth, 1367.5 watts/m$^2$, does not apply to Kerplooie; you will need to use the Inverse Square Law to deduce the energy received per square metre at Kerplooie's distance from the Sun (see Chapter 19-2). You may scale the value for Earth if you wish, as long as you do this correctly. (10)

**Answer:** The rapid rotation means that we may assume that the average day and night temperatures are the same for the purpose of calculating the planetary mean temperature, just as in the case of Earth.

It is simplest to scale the solar constant value for Earth to Kerplooie's orbit: the value is

$1367.5(1.00000/1.27761)^2 = 837.78$ watts/m$^2$ and full credit for this part was given whether this method was used or the solar constant was calculated from $L_{\rm sun}/4\pi a^2$, as long as the result was correct.

The Bond Albedo = 0.27.

Using the formulation in the lectures, the relevant factor is $(1 - A) = 0.73$:

$T_K = \sqrt[4]{\frac{0.73\times837.38}{4\times5.67\times10^{-8}}} = 227.9$ K

The Greenhouse Effect adds 28 degrees to this, so the mean temperature is $T_K = 255.9$ K = -17 C. Kerplooie is a cold planet, but the equatorial region may be comfortably temperate for humans.

Total marks possible for Paper 1: 20