Problem 6.1: The mass estimate from the timing argument involves solving a difficult differential equation and then an algebraic equation. But one can do a back-of-the-envelope version of the calculation using just dimensional analysis.

Show that the inputs (a) the Universe is $\sim 10\,\mathrm{Gyr}$ old and the Milky Way and M31 formed early, (b) M31 is turning round about now, (c) M31 is $\sim 1\,\mathrm{Mpc}$ away, and (d) $G = 4.98 \times 10^{-15}\, M_\odot^{-1}\,\mathrm{pc}^3\,\mathrm{yr}^{-2}$ imply that the combined mass of the Milky Way and M31 is $M \simeq 2 \times 10^{12} M_\odot$. [10]

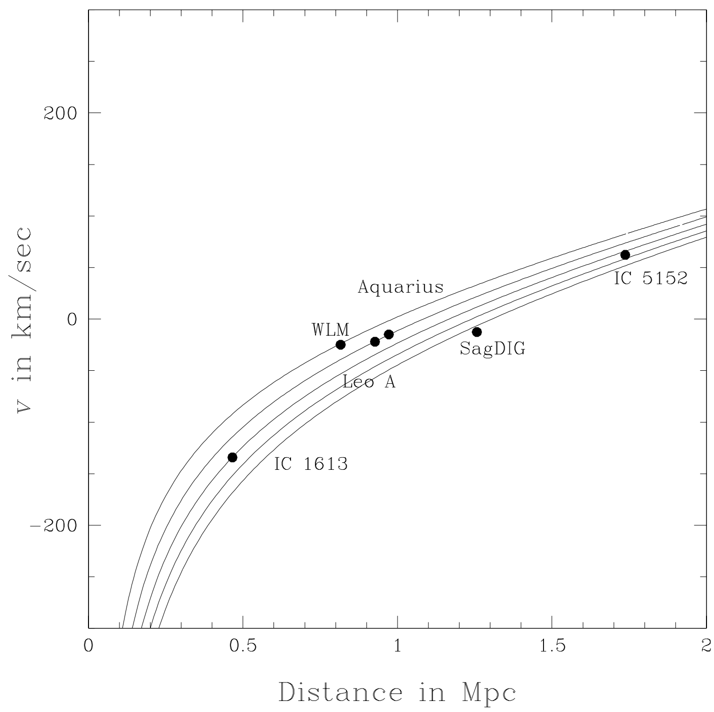

Figure 6.2: Distances and velocities of six Local Group dwarf galaxies, and predictions for different values of GM/τ_0 (by Alan Whiting).

The timing argument can be applied not only to Andromeda, but also to Local Group dwarf galaxies (which have much less mass and behave just as tracers). Figure 6.2 shows plots l against dl/dt for some Local Group dwarfs, along with the predictions of the timing argument for different values of GM/τ_0.

$$\begin{aligned} l &= \left(GM\tau_0^2\right)^{\frac{1}{3}} (1 - \cos\eta). \\ \frac{dl}{dt} &= \left(\frac{GM}{\tau_0}\right)^{\frac{1}{3}} \frac{\sin\eta}{1-\cos\eta} \end{aligned} \tag{6.5}$$

Problem 6.2: If we replace sin and cos in (6.5) with sinh and cosh, the result still satisfies the differential equation (6.1). Verify this, and explain how it relevant to Figure 6.2. [25]

THE SOLAR NEIGHBOURHOOD

The Milky Way is a differentially rotating system. The local standard of rest (LSR) is a system located at the sun and moving with the local circular velocity (which is $\simeq 200\,\mathrm{km/sec}$). The sun has its own peculiar motion of $\simeq 13\,\mathrm{km/sec}$ with respect to the LSR.

The rotation velocity and its derivative at the solar position are traditionally expressed in terms of Oort's constants:

$$\begin{aligned} A &= \tfrac{1}{2}\left(\frac{v_\phi}{R} - \frac{\partial v_\phi}{\partial R}\right), \\ B &= -\tfrac{1}{2}\left(\frac{v_\phi}{R} + \frac{\partial v_\phi}{\partial R}\right). \end{aligned} \tag{6.6}$$

The reason is that A is vanishes for solid body rotation, and can be measured from line of sight velocity data without proper motions (which in the past were hard to measure). But now that we have accurate proper motions from Hipparcos, and hence (combining with ground-based line-of-sight velocities) three-dimensional stellar velocities in the solar neighbourhood, A and B are less important.

If you take the average (three-dimensional) velocity and dispersions of any class of stars in the solar neighbourhood, then $\langle v_R \rangle$ and $\langle v_z \rangle$ turn out to be nearly zero, while $\langle v_\phi \rangle$ is such that $\langle v_\phi \rangle - v_{\mathrm{LSR}}$ is negative and $\propto \sigma_{RR}$. This is known as the 'asymmetric drift' and is nothing but our old friend rotational support versus pressure support. Young stars are almost entirely supported by $\langle v_\phi \rangle$, like the gas that produced them. Older stars pick up increasing amounts of pressure support in the form of σ_{RR}; they then need less v_ϕ to support them, and thus tend to lag behind the LSR. The linear relation can be derived from the Jeans equations, but we won't go through that because you've probably had enough of Jeans equations for now...

When examined in detail using Hipparcos proper motions, the velocity structure in the solar neighbourhood is more complicated than anyone expected. Figure 6.3 shows a reconstruction of the stellar (u, v) (i.e., radial and tangential velocity) distribution in the solar neighbourhood for stars in different ranges of the main sequence.[2] Notice the clumps in the velocity distribution which appear for stars of all ages. (And these are clumps only in velocity space, not in real space.) The idea that there are groups of stars at similar velocities is itself not new—it actually dates from the early proper motion measurements of nearly a century ago. But these 'streams' have generally been interpreted as groups of stars which formed in the same complex and were later stretched in real space over several galactic orbits. The surprising new finding is that the 'streams' are seen for stars of all ages, which indicates a dynamical origin; they seem to be wanting to tell us something interesting about Milky Way dynamics, but as yet we don't know what.

[2] The Schwarzschild ellipsoid and its vertex deviation that you may find in textbooks should now be considered obsolete—they are essentially the result of washing out the structure in Figure 6.3.

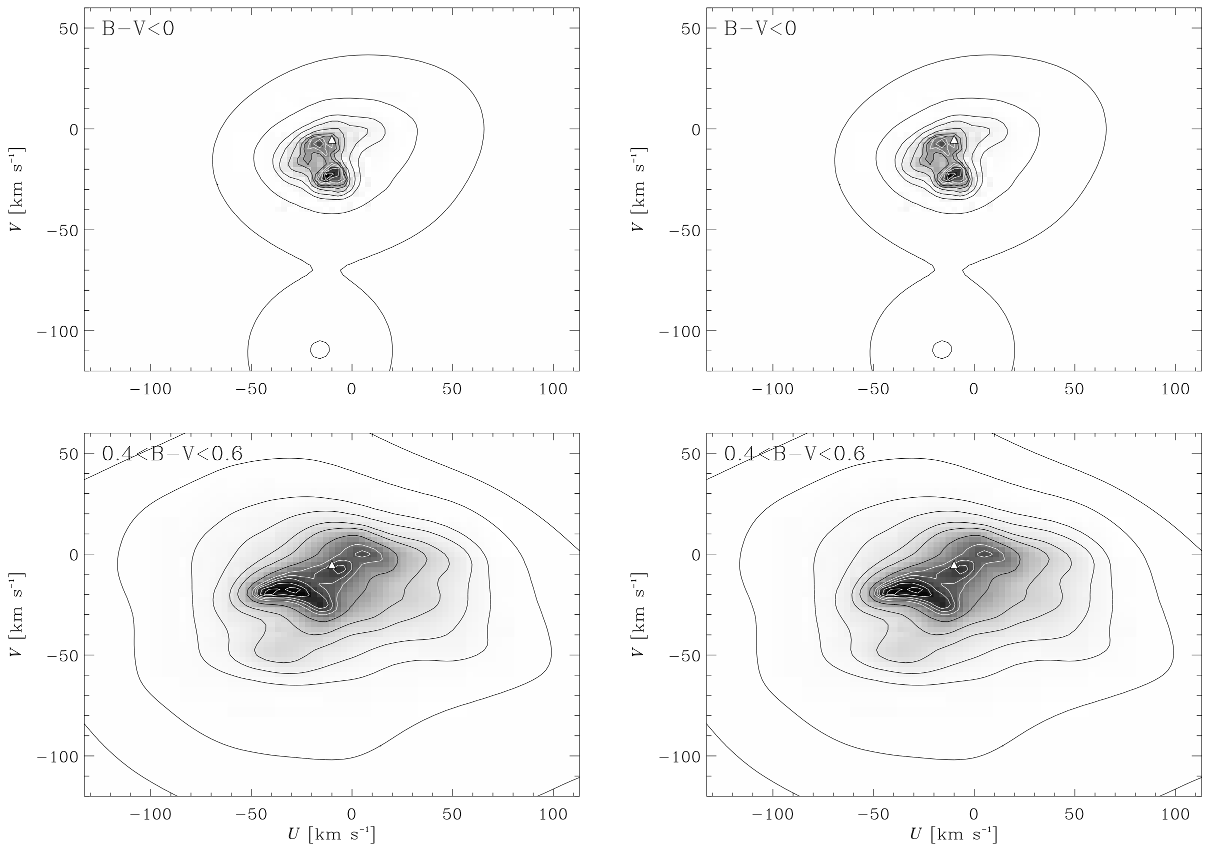

Figure 6.3: Distribution of radial (u) and tangential (v) velocities of main sequence stars in the solar neighbourhood, recently reconstructed from Hipparcos proper motions by Walter Dehnen (1998). The upper left panel is for the youngest (and bluest) stars; these are estimated to be < 0.4 Gyr old. The upper right panel is for stars younger than 2 Gyr, and the lower left panel is for stars younger than 8 Gyr. The lower right panel shows the combined distribution for all main sequence stars. The sun is at $(0, 0)$ and the LSR is marked by a triangle.

The Bar

There is little doubt now that the Milky Way bulge is triaxial—there is a (rotating) bar with the positive l side nearer to us and moving away. The evidence for this was at first indirect, as the following. Consider gas in the ring, which must move on closed orbits. If it moved on circular orbits in the disc, and we measured its Galactic longitude l and line of sight velocity v, then all the gas at positive l would have one sign for v and similarly all the gas at negative l would have the opposite sign for v. In fact gas at positive l is seen with both signs for v, and likewise at negative l. So the gas orbits must be non-circular, and hence the gravitational potential must be non-circular in the disc. This suggests a bar and indeed the observed gas kinematics is well fitted by a bar.

The features of a bar can in fact be seen in an infrared map of the bulge, if you know what to look for. Figure 6.4 shows a bar in the plane, and its effect on an l, b map.

(i) The side nearer to us is brighter. Contours of constant surface brightness are further apart in both l and b on the nearer size.

(ii) Very near the centre, the *further* side appears brighter, so the brightest spot is slightly to the further size of $l = 0$. The reason is that on the further side our