

PHYSICAL CONSTANTS & MATHEMATICAL FORMULAE

Physical Constants

electron charge	$e = 1.60 \times 10^{-19} \text{ C}$
electron mass	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV } c^{-2}$
proton mass	$m_p = 1.673 \times 10^{-27} \text{ kg} = 938.3 \text{ MeV } c^{-2}$
neutron mass	$m_n = 1.675 \times 10^{-27} \text{ kg} = 939.6 \text{ MeV } c^{-2}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J s}$
Dirac's constant ($\hbar = h/2\pi$)	$\hbar = 1.05 \times 10^{-34} \text{ J s}$
Boltzmann's constant	$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1} = 8.62 \times 10^{-5} \text{ eV K}^{-1}$
speed of light in free space	$c = 299\,792\,458 \text{ m s}^{-1} \approx 3.00 \times 10^8 \text{ m s}^{-1}$
permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
Avogadro's constant	$N_A = 6.02 \times 10^{23} \text{ (g-mol)}^{-1}$
gas constant	$R = 8.32 \text{ J (g-mol)}^{-1} \text{ K}^{-1}$
ideal gas volume (STP)	$V_0 = 22.4 \text{ l (g-mol)}^{-1}$
gravitational constant	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Rydberg constant	$R_\infty = 1.10 \times 10^7 \text{ m}^{-1}$
Rydberg energy of hydrogen	$R_H = 13.6 \text{ eV}$
Bohr radius	$a_0 = 0.529 \times 10^{-10} \text{ m}$
Bohr magneton	$\mu_B = 9.27 \times 10^{-24} \text{ J T}^{-1}$
fine structure constant	$\alpha \approx 1/137$
Wien displacement law constant	$b = 2.898 \times 10^{-3} \text{ m K}$
Stefan's constant	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
radiation density constant	$a = 7.55 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$
mass of the Sun	$M_\odot = 1.99 \times 10^{30} \text{ kg}$
radius of the Sun	$R_\odot = 6.96 \times 10^8 \text{ m}$
luminosity of the Sun	$L_\odot = 3.85 \times 10^{26} \text{ W}$
mass of the Earth	$M_\oplus = 6.0 \times 10^{24} \text{ kg}$
radius of the Earth	$R_\oplus = 6.4 \times 10^6 \text{ m}$

Conversion Factors

1 u (atomic mass unit) = $1.66 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV } c^{-2}$	1 Å (angstrom) = 10^{-10} m
1 astronomical unit = $1.50 \times 10^{11} \text{ m}$	1 g (gravity) = 9.81 m s^{-2}
1 eV = $1.60 \times 10^{-19} \text{ J}$	1 parsec = $3.08 \times 10^{16} \text{ m}$
1 atmosphere = $1.01 \times 10^5 \text{ Pa}$	1 year = $3.16 \times 10^7 \text{ s}$

Polar Coordinates

$$x = r \cos \theta \quad y = r \sin \theta \quad dA = r dr d\theta$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$$

Spherical Coordinates

$$x = r \sin \theta \cos \phi \quad y = r \sin \theta \sin \phi \quad z = r \cos \theta \quad dV = r^2 \sin \theta dr d\theta d\phi$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

Calculus

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
x^n	nx^{n-1}	$\tan x$	$\sec^2 x$
e^x	e^x	$\sin^{-1} \left(\frac{x}{a} \right)$	$\frac{1}{\sqrt{a^2-x^2}}$
$\ln x = \log_e x$	$\frac{1}{x}$	$\cos^{-1} \left(\frac{x}{a} \right)$	$-\frac{1}{\sqrt{a^2-x^2}}$
$\sin x$	$\cos x$	$\tan^{-1} \left(\frac{x}{a} \right)$	$\frac{a}{a^2+x^2}$
$\cos x$	$-\sin x$	$\sinh^{-1} \left(\frac{x}{a} \right)$	$\frac{1}{\sqrt{x^2+a^2}}$
$\cosh x$	$\sinh x$	$\cosh^{-1} \left(\frac{x}{a} \right)$	$\frac{1}{\sqrt{x^2-a^2}}$
$\sinh x$	$\cosh x$	$\tanh^{-1} \left(\frac{x}{a} \right)$	$\frac{a}{a^2-x^2}$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$	uv	$u'v + uv'$
$\sec x$	$\sec x \tan x$	u/v	$\frac{u'v-uv'}{v^2}$

Definite Integrals

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \quad (n \geq 0 \text{ and } a > 0)$$

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a^3}}$$

Integration by Parts:
$$\int_a^b u(x) \frac{dv(x)}{dx} dx = u(x)v(x) \Big|_a^b - \int_a^b \frac{du(x)}{dx} v(x) dx$$

Series Expansions

Taylor expansion: $f(a+x) = f(a) + xf'(a) + \frac{x^2}{2!}f''(a) + \dots$

Binomial expansion: $(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$ and $\binom{n}{k} = \frac{n!}{(n-k)!k!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots \quad (|x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, \quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad \text{and} \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\ln(1+x) = \log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \quad (|x| < 1)$$

$$\text{Geometric series: } \sum_{k=0}^n r^k = \frac{1-r^{n+1}}{1-r}$$

Stirling's formula: $\log_e N! = N \log_e N - N$ or $\ln N! = N \ln N - N$

Trigonometry

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

$$\sin 2a = 2 \sin a \cos a$$

$$\cos 2a = \cos^2 a - \sin^2 a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a$$

$$\sin a + \sin b = 2 \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$$

$$\sin a - \sin b = 2 \cos \frac{1}{2}(a+b) \sin \frac{1}{2}(a-b)$$

$$\cos a + \cos b = 2 \cos \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$$

$$\cos a - \cos b = -2 \sin \frac{1}{2}(a+b) \sin \frac{1}{2}(a-b)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) \quad \text{and} \quad \sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

$$\cosh \theta = \frac{1}{2}(e^\theta + e^{-\theta}) \quad \text{and} \quad \sinh \theta = \frac{1}{2}(e^\theta - e^{-\theta})$$

Spherical geometry: $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$ and $\cos a = \cos b \cos c + \sin b \sin c \cos A$

Vector Calculus

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = A_j B_j$$

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \hat{\mathbf{i}} + (A_z B_x - A_x B_z) \hat{\mathbf{j}} + (A_x B_y - A_y B_x) \hat{\mathbf{k}} = \epsilon_{ijk} A_j B_k$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C}) \mathbf{B} - (\mathbf{A} \cdot \mathbf{B}) \mathbf{C}$$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\text{grad } \phi = \nabla \phi = \partial_j \phi = \frac{\partial \phi}{\partial x} \hat{\mathbf{i}} + \frac{\partial \phi}{\partial y} \hat{\mathbf{j}} + \frac{\partial \phi}{\partial z} \hat{\mathbf{k}}$$

$$\text{div } \mathbf{A} = \nabla \cdot \mathbf{A} = \partial_j A_j = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\text{curl } \mathbf{A} = \nabla \times \mathbf{A} = \epsilon_{ijk} \partial_j A_k = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{\mathbf{i}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{\mathbf{j}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{\mathbf{k}}$$

$$\nabla \cdot \nabla \phi = \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

$$\nabla \times (\nabla \phi) = 0 \quad \text{and} \quad \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$