King's College London

UNIVERSITY OF LONDON

M.Sci. EXAMINATION

CP/4750 Image Capture and Sensor Technology

Summer 2002

Time allowed: 3 Hours

Candidates must answer THREE questions.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Answer THREE questions

Planck's constant	h	$= 6.63 \times 10^{-34}$ J s
Boltzmann's constant	k	$= 1.38 \times 10^{-23}$ J K^{-1}
Stefan-Boltzmann's constant	σ	$= 5.67 \times 10^{-8}$ J S^{-1} m^{-2} K^{-4}
Elementary Charge	e	$= 1.60 \times 10^{-19}$ C
Electron charge-to-mass ratio	e/m	$= 1.76 \times 10^{11}$ C kg^{-1}
Vacuum permittivity	ϵ_0	$= 8.85 \times 10^{-12}$ F m^{-1}
Vacuum speed of light	c	$= 3.00 \times 10^{8}$ m s^{-1}
Energy of 1 μm wavelength photon	$\frac{h\nu}{e}$	$= 1.24$ eV
Room temperature		$= 300$ K

1) Briefly explain each of the following terms

a) ideal photon detector,

[2 marks]

b) specific detectivity,

[2 marks]

c) the Weiner-Khintchine relationship.

[2 marks]

The probability $p(n, k)$ of detecting n photons in a pulse of unit time interval, when the average photon arrival rate is k, is governed by the Poisson probability distribution

$$p(n, k) = \frac{(k)^n}{n!} \exp(-k).$$

Show that, for the above distribution,

$$\overline{n^2} - \overline{n}^2 = \overline{n}$$

where $\overline{n}$ is the mean of n.

[5 marks]

Calculate the minimum number of photons, per pulse, required to form a digital signal with a bit error rate of $\leq 10^{-9}$.

[5 marks]

A minimum tolerable signal to noise ratio for a telephone system is 10 dB for a bandwidth of 4.0 kHz. Estimate the minimum optical signal power required to achieve this in an analogue optical transmission system assuming that the only source of noise is *shot* noise, the wavelength used is $\lambda = 1.0$ μm and the detector used has unit efficiency.

[4 marks]

2) With the aid of a suitably labelled diagram, explain the principles of operation of an avalanche photodiode, stating the main advantages and disadvantages of this type of detection device.

[6 marks]

The electron and hole currents ($I_{\rm n}, I_{\rm p}$ respectively) in an avalanche photodiode may be described by the coupled equations

$$\frac{dI_{\rm n}}{dz} = \alpha I_{\rm n} + \beta I_{\rm p}$$

$$\frac{dI_{\rm p}}{dz} = -\alpha I_{\rm n} - \beta I_{\rm p}$$

where z is the distance across the semiconductor junction of width w; while α and β are the field dependent ionisation coefficients for electrons and holes, respectively. Assuming that these ionisation coefficients are equal, show that an avalanche current will occur when

$$\int_0^w \alpha \, dz = 1.$$

[5 marks]

An avalanche photodiode, with a full device current gain Γ and unit quantum efficiency, is used to detect an incident illumination of P Watts. The mean squared current fluctuations in the device obey Poisson statistics such that $\overline{i_N^2} = 2eIB$ for a mean signal current I of bandwidth B. Show that

$$\overline{i_N^2} = 2\frac{P}{h\nu} e^2 \Gamma^2 B \left(2 - \frac{1}{\Gamma}\right).$$

Comment on how $\overline{i_N^2}$ compares to that of an ideal quantum limited photon detector.

[9 marks]

3) Describe the five monochromatic (Siedel) primary lens aberrations in an uncorrected optical imaging system. Explain how each of these aberrations manifests itself as a geometrical distortion in the image plane.

[10 marks]

Describe the implementation of an imaging system which utilises a single element infrared detector. You should illustrate the relative merits of image space, object space and afocal scanning schemes. Explain why the signal-to-noise performances achievable with such imaging systems are intrinsically inferior to those which utilise staring arrays.

[10 marks]

4) A thermal detector element with total thermal energy E at a temperature T has a heat capacity $C = \frac{dE}{dT}$. Using the Boltzmann relationship for the probability of detector element to have an energy $P(E_i) \propto \exp - \left(\frac{E_i}{kT}\right)$, show that the mean squared fluctuations in temperature are given by

$$\overline{\Delta T^2} = \frac{kT^2}{C}.$$

[7 marks]

The detector has a thermal conductance $G = \frac{dP}{dT}$, where P is the incident thermal power, and its frequency response is described by

$$\overline{\Delta T^2}(\omega) = \frac{A}{1 + \omega^2 \tau^2},$$

where $\tau = C/G$ is the time constant of the device and $A = \frac{4kT^2}{G}$ is a constant. Use the standard integral $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$ to show that in a low frequency regime, limited to a bandwidth B, the mean squared temperature fluctuations are given by

$$\overline{\Delta T^2} = \frac{4kT^2}{G} B.$$

[7 marks]

Hence show that the noise equivalent power (P) of such an idealised thermal detector of unit area is given by

$$P = 4\sqrt{\sigma k T^5 B}.$$

[3 marks]

Calculate the specific detectivity (D^*) of the detector when it is at a temperature of $300K$.

[3 marks]

5) Describe blackbody radiation and state Wien's law.

[3 marks]

Explain what is meant by the terms 'background limited detection' and 'noise equivalent temperature change' (ΔT).

[3 marks]

Show that the Noise Equivalent Power, P, for a background limited detector system is given by

$$P = \sqrt{\frac{(2h\nu)(P_{\mathrm{s}} + P_{\mathrm{B}})B}{\eta}}$$

where the symbols have their usual meanings.

[4 marks]

An object generates temperature fluctuations in a background radiation field received from a scene which has a blackbody temperature of 300 K. An image of this object is formed using a thermal imaging system with an angular field of view of 1 milliradian and entrance aperture size of 100 cm^2. A cooled filter is incorporated into the imaging system having peak transmittance at 10 μm with a 10% bandwidth. Given Planck's radiation law

$$dI_\nu = \frac{2h\nu^3 d\nu}{c^2\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)},$$

where the symbols have their usual meanings, calculate a value for ΔT for the imaging system if the bandwidth required is 1 MHz.

[10 marks]