Physical Constants

Permittivity of free space	ε_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi\times10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

Atomic number of aluminium		=	13
Atomic weight of aluminum		≈	27
Reduced Planck constant	$\hbar$	=	$h/2\pi$
Binding energy of hydrogen atom ground state		=	13.6 eV

SECTION A – Answer SIX parts of this section

1.1) Place the following examples of naturally occurring plasmas in order of (a) increasing temperature and (b) increasing electron density: (i) the Solar wind, (ii) the Solar chromosphere and (iii) the Earth's ionosphere. Name one man-made plasma which can be hotter and denser than any of those listed.

[7 marks]

1.2) Define the *electron plasma temperature*, T. If $T = 1.5 \times 10^6\,\mathrm{K}$ determine the corresponding value T_{eV} in electron volts.

[7 marks]

1.3) For a plasma in which the ions are singly charged, derive an expression for the rise in potential caused by a local imbalance between the electron and ion densities. Hence argue that, over appropriate length and time scales, the Solar corona is electrically neutral.

[7 marks]

1.4) Under what conditions is *local thermodynamic equilibrium* suitable for describing a plasma? What determines the populations of the ionic energy levels in such a plasma? Discuss the transition between local thermodynamic equilibrium and coronal equilibrium.

[7 marks]

1.5) The plasma angular frequency ω_{P} is given by

$$\omega_{\mathrm{P}}^2 = \frac{e^2 n_{\mathrm{e}}}{\varepsilon_0 m_{\mathrm{e}}}$$

where n_{e} is the plasma electron density. State the significance of the critical density n_{cr} and use the equation for ω_{P} to define it. Calculate a value for n_{cr} at a wavelength of 355 nm.

[7 marks]

1.6) Briefly discuss, using a flowchart or otherwise, how particle-in-cell codes are used in modelling plasmas.

[7 marks]

1.7) Describe how a plasma that would be suitable for an x-ray laser could be formed.

[7 marks]

1.8) The magnetic moment of a charged particle of mass m spiralling around a field line in a plasma is $\frac{1}{2} m v_\perp^2 \big/ B$ where $v_\perp$ is the velocity component perpendicular to the magnetic flux density **B**. Use this to explain how charged particles are trapped in magnetic confinement fusion experiments. If, at the point of weakest field, $B_0 = 0.8\,\mathrm{T}$ and $v_\perp = \frac{1}{2} v_z$, where v_z is the velocity component in the direction of **B**, determine the value of B at which the particles are trapped.

[7 marks]

SECTION B – Answer TWO questions

2) a) Show, explaining each step, that the equation of motion for a displaced electron in a plasma, assuming that thermal energy and collisions can be neglected, is

$$m_e\ddot{x} + \left(e^2 n_e / \varepsilon_0\right)x = 0,$$

where n_e is the plasma electron density.

[8 marks]

b) By considering the behaviour of the free electrons in a plasma under the influence of an external electromagnetic field, derive an expression for the plasma refractive index n_P.

[12 marks]

c) Calculate a value for n_P at a wavelength of 532 nm for a plasma with an electron density of $10^{27}\,\mathrm{m}^{-3}$. Determine the longest wavelength of radiation that can propagate in this plasma.

[10 marks]

3) a) What is the *principle of detailed balance*?

[2 marks]

b) Define the *Einstein coefficients*. Use the principle of detailed balance and the blackbody formula

$$w_\omega = \frac{n_P^2 \hbar \omega^3}{2\pi^2 c^3} \frac{1}{e^{\hbar\omega/kT} - 1},$$

where w_ω is the spectral energy density and n_P is the plasma refractive index, to derive relationships between the Einstein coefficients.

[12 marks]

c) If two energy levels E_1 and E_2 have statistical weights $g_1 = 6$ and $g_2 = 10$ determine the ratio of number densities n_2/n_1 above which amplification of an incident parallel beam of radiation can occur. State one other condition the incident beam must satisfy in order for amplification to occur.

[6 marks]

d) Amplification of radiation in plasmas can lead to short wavelength lasing action. Figure 3.1 shows spectra obtained from measurements on plasmas formed from a plastic, Mylar. Explain the notation used to label the spectral lines, and identify which, if any, of the labelled lines shows lasing action, explaining your reasoning for each line.

[10 marks]

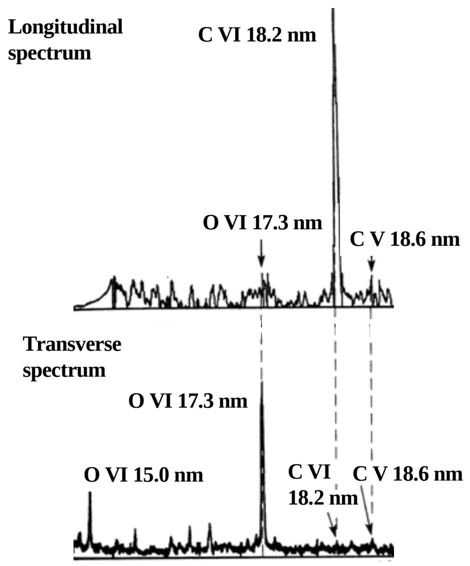

Figure 3.1

4) a) What causes plasma spectral lines to be (i) Doppler broadened and (ii) pressure broadened? Identify two types of pressure broadening.

[4 marks]

b) Show that a Maxwellian distribution $f(v)$ of ion speeds, i.e., $f(v) \propto \exp(-m_{\rm ion}v^2/2k_{\rm B}T)$, gives a contribution to the spectral line shape that has a Gaussian profile. Show that an expression for the full width at half maximum $\Delta_{\rm D}$ (FWHM) of this profile is

$$\Delta_{\rm D} = 2(2\ln 2)^{1/2}\frac{v_{\rm ion}}{c}\nu_0 .$$

[8 marks]

c) In an aluminium plasma the FWHM of a Doppler broadened line at 1.6 keV is measured to be ≈ 0.5 eV. Estimate the ion temperature in electron volts.

[4 marks]

d) Show that, in the nearest neighbour approximation, the profile of a Stark broadened spectral line is proportional to $|\nu - \nu_0|^{-5/2}$, where ν_0 is the central frequency of the line. Discuss why, close to ν_0, the profile deviates from this shape.

[8 marks]

e) Show that the width of a Stark broadened line is approximately proportional to $n_{\rm i}^{2/3}$, where $n_{\rm i}$ is the ion number density. A Stark broadened line in the spectrum of an aluminium plasma with $n_{\rm i} = 10^{16}\,{\rm m}^{-3}$ has a calculated width, in a particular model, of 10 eV. The same line in the spectrum of a nebula has a width of 2 eV. *Estimate*, stating any assumptions, the ion density of the nebula.

[6 marks]

5) a) Describe, with the aid of a suitably labelled energy-level diagram, the processes of *bremsstrahlung* and *recombination radiation*. What is meant by *classical bremsstrahlung*?

[6 marks]

b) The power spectrum for classical bremsstrahlung from an electron of initial speed v may be written as

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} = \frac{Z^2 e^6}{(4\pi\varepsilon_0)^3} \frac{16 n_\mathrm{i}}{3 m_\mathrm{e}^2 c^3 v} \frac{\pi}{\sqrt{3}} G$$

where n_i is the ion density and the other symbols have their usual meanings. Explain the significance of the *Gaunt factor*, G.

[4 marks]

c) Modify the above equation for the bremsstrahlung power spectrum to give an expression for the power emitted in recombination radiation from an electron of initial speed v in a hydrogenic plasma, explaining your reasoning. You may assume that for hydrogenic ions of charge Ze the energy of a bound state with principal quantum number n is given by

$$E_n = -\frac{Z^2 e^4 m_\mathrm{e}}{2(4\pi\varepsilon_0\hbar)^2} \frac{1}{n^2}.$$

[10 marks]

d) For a Maxwellian distribution of electron speeds the ratio of powers emitted in recombination radiation (RR) and bremsstrahlung (B) is given by

$$\frac{\mathrm{RR}}{\mathrm{B}} = -\frac{2E_n}{nT_\mathrm{eV}} \frac{G_n}{\bar{g}(\omega, T_\mathrm{eV})} \exp\left(-\frac{E_n}{T_\mathrm{eV}}\right)$$

where E_n and T_eV (the electron temperature) are in electron volts. To what do the Gaunt factors G_n and $\bar{g}(\omega, T_\mathrm{eV})$ correspond? Use this equation to plot the ratio of intensities in recombination radiation and bremsstrahlung for a plasma consisting of hydrogenic aluminium ions and for electron temperatures of 100 eV and 500 eV and principal quantum numbers of 1, 3 and 6, explaining any assumptions. Discuss your results.

[10 marks]