King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3650 Introductory plasma physics

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light $c = 2.998\times10^{8}\,\mathrm{m\,s^{-1}}$

Boltzmann constant $k = 1.381\times10^{-23}\,\mathrm{J\,K^{-1}}$

Planck constant $h = 6.629\times10^{-34}\,\mathrm{J\,s}$

Proton mass $m_\mathrm{p} = 1.673\times10^{-27}\,\mathrm{kg}$

Electron mass $m_\mathrm{e} = 9.109\times10^{-31}\,\mathrm{kg}$

Atomic weight of carbon ≈ 12

Atomic weight of iron ≈ 56

Electron charge $e = -1.602\times10^{-19}\,\mathrm{C}$

Electron volt, $1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J}$

Permittivity of vacuum $\varepsilon_0 = 8.854\times10^{-12}\,\mathrm{F\,m^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) Argue that, in a plasma with an electron density of $10^{25}\,\mathrm{m^{-3}}$, any local charge imbalance over a distance of the order of 0.1 mm will result in a large rise in potential difference which cannot be maintained.

[7 marks]

1.2) Determine the electron temperature, in electron volts, of a plasma of temperature 10^{6}K. What is the mean electron speed in this plasma? If the plasma consists of ionised carbon, and the electron and ion temperatures are equal, estimate the mean ion speed.

[7 marks]

1.3) The plasma angular frequency ω_P is given by

$$\omega_\mathrm{P}^2 = \frac{e^2 n_\mathrm{e}}{\varepsilon_0 m_\mathrm{e}}$$

where the symbols have their usual meanings. Use this to define the critical density n_cr. State the significance of the critical density and calculate a value for n_cr at wavelength of 248 nm.

[7 marks]

1.4) Briefly discuss how particle-in-cell codes are used in modelling plasmas.

[7 marks]

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1.5) What causes spectral lines to be naturally broadened? The shape of a naturally broadened line with a central frequency ν_0 is given by the Lorentzian profile

$$I(\nu) = I(\nu_0)\frac{1}{1+[2\pi(\nu-\nu_0)\tau]^2}\,.$$

Derive an expression for the full width at half maximum (FWHM) of this profile. The FWHM of a naturally broadened line is measured to be 0.1 eV. What is the lifetime of the upper ionic level?

[7 marks]

1.6) Describe, with the aid of a suitably labelled energy-level diagram, the three main processes by which a plasma emits radiation. Write down one other plasma process via which radiation could be emitted, and state why it is not normally significant.

[7 marks]

1.7) Under what conditions is collisional-radiative equilibrium suitable for describing a plasma? As the electron density is reduced, what type of equilibrium will become appropriate?

[7 marks]

1.8) The magnetic moment of an electron or ion spiralling around a field line in a plasma is $\frac{1}{2}mv_\perp^2 \big/ B$ where $v_\perp$ is the velocity component perpendicular to the magnetic flux density **B**. Use this to explain how plasma electrons and ions are trapped in magnetic confinement fusion. If, at the point of weakest field, $B_0 = 1.5\text{T}$ and $v_\perp = v_z$, where v_z is the velocity component along **B**, determine the value of B at which the particles are trapped.

[7 marks]

SECTION B – Answer TWO questions

2.) a) The rate of change in intensity as radiation passes through a plasma is given by

$$n_{\mathrm{p}}^2 \mathrm{d}\left(\frac{J_\omega}{n_{\mathrm{p}}^2}\right) = \left(-J_\omega a_\omega + j_\omega\right)\mathrm{d}x \qquad (2.1)$$

Define the terms n_{P}, J_ω, a_ω and j_ω.

[7 marks]

b) Obtain solutions of equation (2.1) in the limiting cases of (i) no absorption and (ii) no emission (with constant absorption coefficient), and briefly discuss your solutions.

[8 marks]

c) What is meant by the optical depth, τ_ω, of a plasma? Briefly discuss why the concept of optical depth is useful. By defining a suitable source function rewrite equation (2.1) in terms of the optical depth to give the equation of radiative transfer.

[7 marks]

d) Determine the mean optical depth of an optically thick plasma. Assuming that emission can be ignored and that a_ω is constant, obtain values for a_ω and the mean free path of radiation in the plasma if the intensity drops by a factor of 3 over a distance of 10 µm. What is the optical depth of a 20 µm thickness of this plasma?

[8 marks]

3.) a) What causes plasma spectral lines to be (i) Doppler broadened and (ii) pressure broadened? Identify two types of pressure broadening.

[4 marks]

b) Show that a Maxwellian distribution $f(v)$ of ion speeds, i.e., $f(v) \propto \exp(-m_{\mathrm{ion}}v^2/2kT)$, gives a contribution to the spectral line shape that has a Gaussian profile. Derive an expression for the full width at half maximum (FWHM) of this profile.

[8 marks]

c) In an iron plasma the FWHM of a Doppler broadened line at 6.7 keV is measured to be ≈ 1.5 eV. Estimate the ion temperature in electron volts.

[4 marks]

d) Show that, in the nearest neighbour approximation, the profile of a Stark broadened spectral line with central frequency ν_0 is proportional to $|\nu - \nu_0|^{-5/2}$,. Discuss why, close to ν_0, the profile deviates from this shape.

[8 marks]

e) Show that the width of a Stark broadened line is approximately proportional to $n_{\mathrm{i}}^{2/3}$, where n_{i} is the ion number density. A Stark broadened line in the spectrum of an aluminium plasma with $n_{\mathrm{i}} = 10^{15}\,\mathrm{m}^{-3}$ has a calculated width, in a particular model, of 2 eV. The same line in the spectrum of a nebula has a width of 10 eV. *Estimate*, stating any assumptions, the ion density of the nebula.

[6 marks]

4.) a) Draw a labelled sketch showing the variation in temperature from the solar surface to the outer regions of the corona.

[4 marks]

b) The electron temperature of the corona is $T_e \approx 100\,\text{eV}$. Describe, with reference to the equilibrium conditions, how this was determined.

[8 marks]

c) Draw a labelled diagram showing the variation of the intensity of the coronal components as a function of distance from the Sun, out to a distance of about five solar radii. Use your diagram to explain why the corona is best observed during a total solar eclipse.

[8 marks]

d) In a one-dimensional plasma the ion number density n_i and speed v_i are related by

$$m_i n_i \left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x} \right) v_i = -\gamma \left(Z T_e + T_i \right) \frac{\partial n_i}{\partial x} \tag{4.1}$$

where $\gamma = 5/3$ is the ratio of specific heats. Which plasma model is used to derive this equation? In which situation is it valid? Use equation (4.1) to obtain an expression for the expansion speed of the plasma, stating any assumptions.

[5 marks]

e) Estimate the expansion speed of the solar corona. What does the expanding corona cause? Explain any discrepancy between the calculated speed and the measured expansion speed of $\approx 500\,\text{km}\,\text{s}^{-1}$.

[5 marks]

5.) a) Explain what is meant by an amplified spontaneous emission (ASE) laser. How could a plasma suitable for such a laser in the soft x-ray region be formed?

[5 marks]

b) Draw a labelled diagram to demonstrate *one* pumping mechanism used in soft x-ray lasers.

[4 marks]

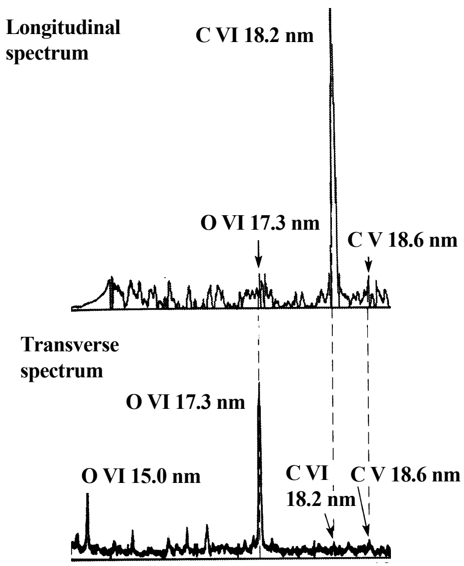

Figure 5.1

c) Figure 5.1 shows spectra obtained from measurements on plasmas formed from a plastic, Mylar. Explain the notation used to label the spectral lines, and identify which, if any, of the lines shows lasing action, explaining your reasoning for each line.

[7 marks]

d) The output of an ASE laser can be described by the equation

$$I = \frac{J_\mathrm{s}}{G}\frac{\left(\mathrm{e}^{GL}-1\right)^{3/2}}{\left(GL\mathrm{e}^{GL}\right)^{1/2}} \tag{5.1}$$

where J_s describes the spontaneous emission strength. Why is equation (5.1) not a simple exponential, i.e., $I \propto \mathrm{e}^{GL}$?

A minimum value of 5 for the gain-length product is normally taken to mean that significant lasing action has taken place. By what factor would the output of the ASE laser increase if the length of the plasma was then doubled, from x to $2x$? If another plasma medium, of the same length $2x$ but double the gain, was used, by what factor would the output increase further?

[6 marks]

e) Why is Doppler broadening normally the dominant line-broadening mechanism in plasmas suitable for ASE lasers? The gain coefficient for a Doppler-broadened line is given by

$$G = \frac{\lambda^3}{16\pi(2\ln 2)^{1/2}}\left(\frac{m_\mathrm{i}}{T_\mathrm{i}}\right)^{1/2} A_{\mathrm{u}\ell}n_\mathrm{u}F \tag{5.2}$$

Define the terms $A_{\mathrm{u}\ell}$, n_u and F. Use equation (5.2) to obtain an expression for the minimum required pumping rate. Briefly discuss the implications for short wavelength lasers suggested by equation (5.1) and by the pumping rate expression.

[8 marks]

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