King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3640 Modern Topics in Astrophysics

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light, $c = 3.00\times10^{8}\,\mathrm{m\,s^{-1}}$

Solar mass, $M_{\odot} = 1.99\times10^{30}\,\mathrm{kg}$

Proton mass, $m_{\mathrm{p}} = 1.67\times10^{-27}\,\mathrm{kg}$

Electron volt, $1\,\mathrm{eV} = 1.60\times10^{-19}\,\mathrm{J}$

1 parsec, $1\,\mathrm{pc} = 3.09\times10^{16}\,\mathrm{m}$

1 year, $1\mathrm{y} = 3.16\times10^{7}\,\mathrm{s}$

SECTION A. Answer any **SIX** parts of this section.

1.1 State the cosmological principle, and state what the principle implies about the structure of the Universe. Use the principle to deduce the Hubble Law in terms of the Hubble parameter H and the length scale R.

[7 marks]

1.2 The Friedman equation can be written in the form

$$\left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 + \frac{kc^2}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}.$$

Briefly explain the origin of this equation, and explain the physical significance of the terms k, ρ and Λ.

[7 marks]

1.3 The cosmic microwave background radiation has a present temperature of 2.735 K and is considered to be the relic of a radiation field at an earlier time t_1 at a temperature of 3000 K. What physical event occurred at t_1? Estimate the time t_1.

[7 marks]

1.4 State briefly the cosmological significance of the amounts of deuterium and lithium present in the Universe.

[7 marks]

1.5 Define *helicity*. Argue that, if neutrinos have zero rest mass, then neutrinos and antineutrinos must have fixed helicities.

[7 marks]

1.6 State, with reasons, the main properties which detectors of supernova neutrinos should possess. Discuss any differences between these properties and those needed for the detection of solar neutrinos. *You should not refer to any particular detectors.*

[7 marks]

1.7 Imagine that a supernova is first observed through its optical output at 00.01 (Universal Time) on 1 January 2000. On subsequent analysis of data recorded by a neutrino detector, the neutrino pulse emitted by the supernova arrived 7 days earlier, at 00.01 (UT) on 25 December 1999. *Estimate*, stating any assumptions, the radius of the collapsing star.

[7 marks]

1.8 From the analysis of geochemical samples, it is deduced that the rate of solar neutrino production has decreased from that predicted by the standard solar model (SSM) to the current detection rate (about 50% of SSM) over a period of about 10^5 years. Briefly discuss a possible explanation of this observation.

[7 marks]

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SECTION B. Answer any **TWO** questions from this section.

2. a) Calculate the value of $H_0 t_0$ given that $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $t_0 = 14\times10^9$ years.

[4 marks]

b) One way to account for values of $Ht > 1$ is to include the cosmological constant Λ in the Friedmann equation (question 1.2). Solve the Friedmann equation, including Λ, in the case of a flat, matter-dominated universe, and show that

$$(\Lambda/3)^2 R^3 = \mu \sinh^2\theta$$

where

$$\theta = \frac{3}{2}\left(\frac{1}{3}\Lambda\right)^{1/2} t$$

and μ is a constant. Calculate H and Ht according to this model and show that $\theta_0 \approx 1.3$, where θ_0 is the value of θ at the present epoch.

[14 marks]

c) Show that, according to this model, the present value of Ω_M (the ratio of the observed density of the Universe to the closure density) is about 0.26. How does this value agree with values of Ω_M deduced from other evidence?

[12 marks]

3. Write an essay on the evolution of the Universe to the present time according to current ideas about the Big Bang. Include an emphasis on inflation and the problems inflation attempts to solve.

[30 marks]

4. a) Describe the process which gives rise to the thermal neutrino burst during a supernova explosion, explaining why neutrinos and antineutrinos of all types — electron (e), muon (μ) and tau (τ) — are produced.

[5 marks]

b) Assuming that an Earth-based detector can detect all types of neutrino and antineutrino, explain why the observed energy distributions are not purely thermal and why those of e, μ and τ neutrinos would be different.

[7 marks]

c) Describe the process which gives rise to the neutronisation neutrino burst and compare the expected numbers of neutrinos emitted in the neutronisation and thermal bursts.

[6 marks]

d) A supernova explosion is determined to be due to a star 25 light years from the Earth. Estimate the number of neutrinos (per square metre) incident at the Earth from the neutronisation burst.

[2 marks]

e) The average energy of the observed neutrinos decreases from 25 MeV to 12 MeV over a period of 10 seconds. Estimate, stating any assumptions, the neutrino rest mass. Explain why this estimate is an upper limit, and compare the limit with that obtained from other experiments.

[10 marks]

5. a) Briefly describe the calculation of neutrino fluxes in the *Standard Solar Model*.

[9 marks]

b) The currently favoured explanation of the solar neutrino problem is associated with neutrino oscillations in matter. Qualitatively explain this statement.

[8 marks]

c) Discuss the processes which give rise to neutrino production in the Earth's atmosphere. Show that, in the absence of oscillations, about twice as many low-energy muon neutrinos (and antineutrinos) as electron neutrinos (and antineutrinos) would be incident at the Earth's surface. How would this ratio change at higher energies?

[7 marks]

d) The measured ratio $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ is lower than expected. Discuss the interpretation of this result and any bearing it has on the solar neutrino problem.

[6 marks]

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