King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3640 Modern Topics in Astrophysics

Summer 1997

Time allowed: THREE Hours

**Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.**

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Gravitational constant	$G = 6.7\times10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Mass of proton	$m_p = 1.67\times10^{-27}$ kg
Solar mass	$M_\odot = 1.99\times10^{30}$ kg
Speed of light in vacuum	$c = 3\times10^{8}\,\mathrm{m\,s^{-1}}$
1 Year	3.16×10^{7} s
Cross section for neutrino–electron scattering	$9.3\times10^{-49}E_\nu\,\mathrm{m}^2$ (E_ν in MeV)
Cross section for antineutrino–proton scattering	$7.5\times10^{-48}E_{\bar{\nu}}^2\,\mathrm{m}^2$ ($E_{\bar{\nu}}$ in MeV)

SECTION A. Answer any **SIX** questions.

1.1 State the cosmological principle, and state what the principle implies about the structure of the universe. Use the principle to deduce the Hubble Law in terms of the Hubble parameter H and the length scale R.

[7 marks]

1.2 The Friedmann equation is

$$\left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 + \frac{kc^2}{R^2} = \frac{8\pi G}{3}\rho.$$

Define the closure density ρ_c and deduce an expression for $|\Omega - 1|$ in terms of H and R, where $\Omega = \rho/\rho_c$. If $H_0 = 80\,\mathrm{km\,s^{-1}Mpc^{-1}}$ what is the value of ρ_c (in $\mathrm{kg\,m^{-3}}$) at the present epoch?

[7 marks]

1.3 What are the properties of the cosmic microwave background radiation, and how did it originate?

[7 marks]

1.4 State briefly the cosmological significance of measurements of the amount of deuterium present in the universe.

[7 marks]

1.5 Parity is the operation which inverts all space axes. Assuming that the neutrino rest mass m_ν is zero, explain why the quantum number associated with the parity operation cannot be conserved by weak interactions.

[7 marks]

1.6 Explain why, in a supernova explosion, it is energetically favourable for protons in the star's core to be converted into neutrons. *Estimate* the number of neutrinos released during this process.

[7 marks]

1.7 Explain qualitatively how a black hole at the centre of the Sun could reduce the emitted neutrino flux. Why is this generally considered to be an unlikely explanation of the solar neutrino problem?

[7 marks]

SEE NEXT PAGE

1.8 The solar neutrino problem could be due to a non-zero neutrino magnetic moment. Explain this statement, and describe how the solar neutrino detection rate may then be expected to vary with time.

[7 marks]

SECTION B. Answer **TWO** questions.

2. Calculate the value of $H_0 t_0$ given that $H_0 = 80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $t_0 = 15\times10^9$ years.

[4 marks]

Consider an open vacuum dominated universe with $k = -1$ for which the Friedmann equation is modified to

$$\left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \frac{c^2}{R^2} = \Lambda$$

where Λ is the vacuum energy or cosmological constant. Show that for this model, with the boundary condition that $R = 0$ at $t = 0$,

$$R = c\Lambda^{-1/2}\sinh\left(\Lambda^{1/2}t\right).$$

[12 marks]

[Hint: make the substitution $\Lambda^{1/2}R = c\sinh\theta$.]

Show that this model has the property that Ht is always greater than unity for $t > 0$, and obtain an estimate of $\Lambda^{1/2}$ given that the solution of $x\coth x = 1.227$ is $x \approx 0.844$.

[10 marks]

What feature of the solution of this model makes it unrealistic as a model of the universe now?

[4 marks]

3. Write an essay on the thermal history of the universe in the context of the standard, FRW, model giving details of the important stages in the evolution of the universe.

[30 marks]

4. The supernova SN1987a occured in a star 150,000 light years from the Earth. Twelve interactions ascribed to the production of neutrinos and antineutrinos in this supernova explosion were observed in the Kamiokande II water Cherenkov detector in operation at the relevant time. The detector had a fiducial volume of 2140 tonnes.

a) Briefly describe the operation of the Kamiokande II detector.

[5 marks]

b) Most of the interactions were considered to be due to antineutrinos, rather than neutrinos. State the reasons for this, and draw a Feynman diagram describing the dominant detection process.

[6 marks]

c) The Kamiokande detector has subsequently been upgraded, and is now called Superkamiokande with a fiducial volume of 50,000 tonnes. How many interactions would you expect to observe in this detector if a star *S* with five times the mass of the progenitor of SN1987a and at a distance of 2000 light years exploded as a supernova? Approximately how many of these interactions would be due to neutrinos rather than antineutrinos?

[6 marks]

d) The mean detected energy from the explosion of *S* is observed to decrease from an initial value of 20 MeV to 11 MeV fifteen seconds later. Use this information to obtain a value for the neutrino rest mass m_ν, stating your assumptions.

[8 marks]

e) Explain why this value of m_ν should be treated as an upper limit, and why it is a higher limit than that of $\approx 30\ \mathrm{eV\,c^{-2}}$ obtained from SN1987a.

[5 marks]

SEE NEXT PAGE

5. To date, four experiments have provided data on the flux of neutrinos emitted by the Sun. One of these uses chlorine (^{37}Cl) as the detecting medium, one uses water and two use gallium (^{71}Ga). All of them detect fewer neutrinos than predicted by the Standard Solar Model – the solar neutrino problem.

a) Briefly describe the processes by which neutrinos are produced in the Sun according to the standard solar model. A detailed list of interactions and decays is not required.

[5 marks]

b) What is the main advantage of the gallium detectors compared with the chlorine and water detectors? Briefly describe the mode of operation of one of the gallium detectors.

[6 marks]

c) Comparison of the results from the gallium detectors with those from the chlorine and water detectors appears to indicate that there are very few neutrinos, if any, produced in the Sun by the decay of ^{7}Be. Explain this statement, which is referred to as the solar neutrino paradox.

[7 marks]

d) State a necessary condition for neutrino oscillations to occur. Describe, without a detailed mathematical analysis, how oscillations enhanced by the presence of matter could explain both the solar neutrino problem and the solar neutrino paradox.

[7 marks]

e) Briefly describe the main properties of the BOREXINO detector, currently under construction, which should provide direct data on the flux of ^{7}Be neutrinos and hence help to resolve the solar neutrino paradox.

[5 marks]

SEE NEXT PAGE