King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3480 Electromagnetism and Optics 2

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) State Maxwell's equations for an electromagnetic field propagating in a linear medium of relative permittivity ϵ_r, relative permeability μ_r and electrical conductivity σ, defining all of the symbols used.

[7 marks]

1.2) State the *general wave equation* which applies to an electromagnetic plane wave propagating in a homogeneous linear medium of permittivity ϵ, permeability μ and electrical conductivity σ. What are the two forms of this equation that are appropriate (a) for a dielectric and (b) for a metal?

[7 marks]

1.3) Define the *Poynting vector* and state its units.

Consider an electromagnetic field for which the electric vector is of the form $\mathbf{E} = \mathbf{E}_0\, e^{i\omega t}$ and the magnetic vector is $\mathbf{H} = \mathbf{H}_0\, e^{i\omega t}$ where $\mathbf{E}_0$ and $\mathbf{H}_0$ depend only on the spatial co-ordinates. Show that the *time averaged* Poynting vector is given by $\langle \mathbf{\Pi} \rangle = \frac{1}{2}\, Re\, (\mathbf{E}_0 \times \overline{\mathbf{H}}_0)$ where the symbol $\overline{\mathbf{H}}_0$ represents the complex conjugate of $\mathbf{H}_0$.

[7 marks]

1.4) Define the terms *electromagnetic momentum density* and *electromagnetic radiation pressure* in relation to an electromagnetic field for which the displacement vector is $\mathbf{D}$ and the magnetic induction is $\mathbf{B}$.

Calculate the radiation pressure exerted on a perfectly absorbing surface by the laser beam produced by a 10 mW laser if the cross-sectional area of the beam is $1\,\mathrm{mm}^2$ and the beam exists in free space.

[7 marks]

1.5) Explain, with the aid of a diagram, why it is necessary for the light source used to record an off-axis Fresnel hologram to be both spatially and temporally coherent.

[7 marks]

1.6) Identify three problems that adversely affect the use of a Gabor in-line hologram in forming a 3-dimensional image of an object.

[7 marks]

1.7) Define the term *Pupil Function* as it applies to a simple lens-based imaging system that uses coherent illumination. State the mathematical relationship between the *Pupil Function* and the *Optical Transfer Function* for the same system when using incoherent illumination.

[7 marks]

1.8) State the Rayleigh criterion for determining when the images of two point-like objects are just resolved by an optical system that uses incoherent illumination.

Use the Rayleigh criterion to estimate the resolution of the optical system that has a numerical aperture of 0.8, and the illumination is of wavelength 500 nm.

[7 marks]

SECTION B – Answer TWO questions

2) A plane electromagnetic wave is incident from medium 1 of refractive index n_1 onto the plane surface of medium 2 whose refractive index is n_2. The relative permeabilities of the two media are the same.

Draw a ray diagram illustrating the sign convention used for the directions of the electric and magnetic vectors in the incident, reflected and transmitted wave fields when the electric vector is parallel to the plane of incidence.

[4 marks]

It may be assumed that for each of the three wave fields, the ratio $E/H = \eta$ where η is the wave impedance appropriate to the propagation medium. If θ_i is the angle between the wave normal of the incident wave and the normal to the interface between the two media, use the wave impedance formulas and the boundary conditions for the electric and magnetic field components to show that the reflection coefficient of the boundary for the case when the electric vector lies in the plane of incidence is

$$R_{||} = \frac{n^2 C - \sqrt{n^2 - S^2}}{n^2 C + \sqrt{n^2 - S^2}},$$

where n is the relative refractive index n_2/n_1, $S = \sin\theta_i$ and $C = \cos\theta_i$.

[12 marks]

Sketch a graph showing the magnitude of reflection coefficient $R_{||}$ as a function of the angle of incidence for the case $n < 1$, marking the Brewster and the critical angles on your sketch.

[6 marks]

Define, and obtain expressions for, the Brewster angle θ_B and the critical angle θ_c. Calculate these angles for the boundary between free space and flint glass considering the two cases

a) the wave is incident from free space and

b) the wave is incident from within the flint glass.

Take the optical constants of flint glass as: refractive index = 3.00; conductivity negligible and relative permeability unity.

[8 marks]

3) Derive *Poynting's theorem* for a volume v of free space which contains a source of electromagnetic power P_G.
You may assume that for any two vector fields $\mathbf{X}$ and $\mathbf{Y}$

$$\nabla \cdot \mathbf{X} \times \mathbf{Y} = \mathbf{Y} \cdot \nabla \times \mathbf{X} - \mathbf{X} \cdot \nabla \times \mathbf{Y}$$

[8 marks]

Discuss the interpretation of Poynting's theorem.

[6 marks]

Define the term *radiation zone* as used in the theory of antennas.

[2 marks]

Consider an antenna of current moment $I\,\delta\mathbf{z}$ where $I = I_0\,e^{i\omega t}$ and I_0 is a constant current. The antenna is located in free space at the origin of spherical polar co-ordinate system (r, θ, ϕ).
It is given that, in the radiation zone, the magnetic field strength produced by the antenna has only a ϕ component given as

$$H_\phi = i\frac{k\,I\,\delta z}{4\pi}\frac{e^{-ikr}}{r}\sin\theta,$$

where $k = \omega\sqrt{\mu_0\epsilon_0}$.
Use Maxwell's curl $\mathbf{H}$ equation to show that the θ-component of the electric field in the radiation zone is

$$E_\theta = i\frac{k^2\,I\,\delta z}{4\pi\omega\epsilon_0}\frac{e^{-ikr}}{r}\sin\theta.$$

It may be assumed that the θ-component of curl $\mathbf{H}$ is

$$(\nabla \times \mathbf{H})_\theta = -\frac{1}{r}\frac{\partial(rH_\phi)}{\partial r}.$$

[4 marks]

Hence find an expression for the time-averaged Poynting vector in the radiation zone.

[6 marks]

What is the *state of polarisation* of the field produced by this antenna?
Suggest a method of producing a *circularly* polarised wave field using two such antennas.

[4 marks]

4) Explain briefly what is meant by the term *phase contrast imaging.*

[4 marks]

A coherent monochromatic plane wave illuminates a specimen with transmission function of the form $f(x) = \exp\left[i\phi_0 \cos\left(2\pi x/d\right)\right]$ where ϕ_0 and d are constants, $\phi_0 \ll 1$, and x is a coordinate perpendicular to the optical axis. The specimen is imaged by a system of two identical lenses, each of focal length 40 mm, that are mounted coaxially a distance 80 mm apart. The specimen is placed in the front focal plane of the first lens, and in the common focal plane between the lenses there is a mask with complex amplitude transmission function

$$P(u) = \begin{cases} i & \text{if } u > 0, \\ 1 & \text{if } u = 0, \\ -i & \text{if } u < 0. \end{cases}$$

where $i^2 = -1$, and u is a spatial frequency parallel to the x direction. Assuming the optical system has unit magnification, show that the intensity distribution in the image plane is given approximately by the expression

$$I(x) = 1 + \phi_0^2 \sin^2\left(2\pi x/d\right)$$

stating clearly any other assumptions that you have made in carrying out the calculation. Show that the intensity contrast in the image plane is approximately $\phi_0^2/2$.

[12 marks]

Describe how the mask $P(u)$ referred to above could be realised in practice, and suggest two disadvantages of using such a mask for phase contrast imaging.

[8 marks]

When the illuminating wavelength is 560 nm, the minimum period d that can be successfully imaged by the optical system is 2.4 μm. Calculate the minimum possible diameter of the first lens, assuming that there is free space between the specimen and this lens.

[6 marks]

5) Laser operation requires that the rate of stimulated emission exceeds the rate of resonant absorption. Show that this requires the existence of a *population inversion* within the lasing medium. Briefly explain why it is not generally practicable to sustain continuous light emission from a 3-level laser.

[7 marks]

A photon produced by stimulated emission has the same phase, polarisation, frequency and direction of travel as the stimulating photon. Explain why these properties are not sufficient to ensure that a laser produces coherent monochromatic light output, and describe how the use of a resonant cavity can help to produce this form of output.

[7 marks]

If the laser output is also required to be linearly polarised, explain how Brewster windows could be used to achieve this, and suggest one reason why this method is preferable to the use of a simple polarising filter on the light emitted by the laser.

[6 marks]

A semi-conductor laser has a resonant cavity that is 0.31 mm long, and the laser emits infra-red light with a mean wavelength in free space of 1.55 μm. The refractive index of the semiconducting material at this wavelength is 2.5. Calculate the frequency separation of successive resonant longitudinal modes that could be sustained by the laser.

[5 marks]

If the frequency spread of a single resonant longitudinal mode from this laser is 50 MHz, estimate the coherence time and the coherence length for the laser when it is operated in single mode form.

[5 marks]