King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3402 Solid State Physics

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper. Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Fundamental constants

Planck constant	h	=	6.626×10^{-34} J s
Elementary charge	e	=	1.602×10^{-19} C
Rest mass of an electron	m_e	=	9.109×10^{-31} kg
Permittivity of free space	ε_0	=	8.854×10^{-12} F m^{-1}
Boltzmann constant	k_B	=	1.381×10^{-23} J K^{-1} = 8.617×10^{-5} eV K^{-1}
Avogadro number	N_A	=	6.022×10^{23} mol^{-1}

SECTION A - answer six parts of this section

1.1) Describe the NaCl and CsCl crystal structures.

Explain why some alkali halides have the NaCl structure while others have the CsCl structure.

[7 marks]

1.2) Explain how the Miller indices (hkl) are calculated for a crystal plane.

Hence write down (hkl) for a face, the face diagonal plane and the cube diagonal plane of a crystal which has cubic symmetry.

[7 marks]

1.3) The heat capacity C_v of a metallic crystal at low temperature T may be written as $C_v = \gamma T + \beta T^3$.

What is the origin of these two contributions to C_v? Explain how experimental data obtained in this temperature region may be plotted to obtain a straight line graph.

[7 marks]

1.4) Explain the origin of the Hall effect. A rectangular sample has sides parallel to the coordinate axes x, y, and z. Show that when the sample carries a current density j_x and is placed in a magnetic flux density B_z, an electric field $E_y = -j_x B_z / ne$ is developed, where n is the electron concentration.

[7 marks]

1.5) Explain, with the aid of $E-k$ diagrams, what is meant by the terms *phonon absorption* and *phonon emission* for the optical excitation of an electron from the top of the valence band to bottom of the the conduction band in an indirect-gap semiconductor.

[7 marks]

1.6) Draw a graph illustrating how the concentration of electrons in a typical sample of n-type silicon varies over the temperature range 20 – 1000 K. Label the intrinsic and extrinsic regions of the plot, and indicate the gradients expected in each case.

[7 marks]

1.7) The rectifier equation for the current I flowing through an ideal p-n junction at temperature T and forward bias V is $I = I_0[\exp(eV/k_B T) - 1]$.

Sketch this relationship for both negative and positive V, and define the meaning of the term I_0. Explain why the behaviour of a real diode departs rapidly from the above expression for increasing positive values of V.

[7 marks]

1.8) Explain what is meant by the *Meissner effect* for a type I superconductor.

Show that such a material behaves as a perfect diamagnet.

[7 marks]

SECTION B - answer two questions

2.) Describe the Debye-Scherrer method for studying the X-ray diffraction from polycrystalline powders or wires.

[8 marks]

Show that, for an allowed reflection at angle θ, the diameter of the "ring" observed on the X-ray film is $4R\theta$ where R is the radius of the camera.

[10 marks]

Scattering from the (hkl) planes of a crystal with the diamond structure, and lattice constant a, occurs at angles θ given by $\sin^2\theta = (\lambda^2/4a^2)(h^2 + k^2 + l^2)$ for X-rays with wavelength λ provided (i) h, k and l are all odd or (ii) h, k and l are all even with $(h + k + l)/2$ also even.

Calculate the diameters of one ring in each of the categories (i) and (ii) when R = 57.3 mm, λ = 0.1542 nm and a = 0.3567 nm.

[12 marks]

3.) Explain what is meant by the Fermi energy for a metal crystal.

[6 marks]

Starting from the Schrödinger equation, show that the Fermi energy for N free electrons in a metal crystal of volume V is given by

$$E_{\mathrm{F}} = \frac{h^2}{8m_{\mathrm{e}}}\left(\frac{3N}{V\pi}\right)^{2/3}$$

[16 marks]

Hence show that the Fermi energy of copper is approximately 7 eV.
Copper is monovalent with a density of 8930 kg m^{-3} and an atomic mass number of 63.5.

[8 marks]

4.) By considering the effect of an electric field on a wavepacket with energy E and wavenumber k, show that the effective mass m^* of an electron moving in a periodic potential is

$$m^* = \hbar^2 \Big/ \frac{d^2E}{dk^2}$$

[15 marks]

Describe, paying attention to the experimental conditions necessary, how effective masses in a semiconductor can be measured using cyclotron resonance. Show that resonance is obtained at a frequency $f = eB / 2\pi m^*$ for a magnetic flux density B.

[10 marks]

The light and heavy holes in silicon have effective masses of 0.16 m_{e} and 0.52 m_{e} respectively. Calculate the flux densities at which resonance is observed in a microwave cavity operating at 24 GHz.

[5 marks]

5.) Explain what is meant by a depletion layer at an abrupt p-n junction.

[6 marks]

The depletion layer extends from $x = -w_p$ in the p region to $x = +w_n$ in the n region. If N_a and N_d are the effective acceptor and donor concentrations in the p and n regions, respectively, write down an equation for charge neutrality.

[4 marks]

Hence show that the total width of the depletion layer is

$$d = \left(\frac{2\varepsilon_r \varepsilon_0 V_0}{e(N_a + N_d)} \right)^{1/2} \left[\left(\frac{N_a}{N_d} \right)^{1/2} + \left(\frac{N_d}{N_a} \right)^{1/2} \right]$$

You may assume that the potential in the depletion region is given by

$$V(x) = \left(\frac{eN_a}{2\varepsilon_r \varepsilon_0} \right) (x + w_p)^2 \quad \text{for } -w_p < x < 0$$

$$V(x) = V_0 - \left(\frac{eN_d}{2\varepsilon_r \varepsilon_0} \right) (x - w_n)^2 \quad \text{for } 0 \le x < w_n$$

where ε_r is the relative permittivity and V_0 is the built-in potential.

[10 marks]

The capacitance of a silicon p-n junction of area 10^{-6} m^2 is used with a 100 μH inductor to create a resonant circuit. The junction has $N_a = N_d = 10^{22}$ m^{-3}, $\varepsilon_r = 11.7$ and $V_0 = 0.68$ V.

Calculate the change in the resonant frequency when the bias applied to the junction changes from –1 to –10 V.

For a junction biased with V volts, V_0 must be replaced by $(V_0 - V)$ in the above expression. The resonant frequency of an LC circuit is $1/[2\pi\sqrt{(LC)}]$.

[10 marks]

FINAL PAGE