King’s College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3402 Solid State Physics

Summer 1997

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper. Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Boltzmann constant	k_B	=	8.617×10^{-5} eV K^{-1}
Speed of light	c	=	2.998×10^{8} m s^{-1}
Atomic mass unit	m_u	=	1.661×10^{-27} kg

SECTION A - answer six parts of this section

1.1) Describe the sodium chloride crystal structure.

Explain why KCl (which has the NaCl structure) scatters X-rays as though it had the simple cubic structure with half the true lattice spacing.

[7 marks]

1.2) Show that, for long wavelengths, the velocity of longitudinal waves in a crystal is $v_L = (C/\rho)^{½}$ where C is an appropriate elastic modulus and ρ is the density.

[7 marks]

1.3) Sketch the heat capacity as a function of temperature, as given by the Debye equation for a non-metallic crystal. How do these results differ from those given by the Einstein theory?

[7 marks]

1.4) For the free electrons in a metal, sketch the distribution $n(\varepsilon, T)$ of occupied energy levels as a function of energy ε, at temperature $T = 0$. Explain why only a small fraction of these electrons are thermally excited at room temperature.

[7 marks]

1.5) Explain, without derivation, the concept which allows semiconductor-physicists to avoid dealing with electrons having a negative effective mass.

[7 marks]

1.6) Explain what is meant by a ‘substitutional impurity’ in a group IV semiconductor. Calculate the effective Bohr radius of the wavefunction associated with a typical donor impurity in silicon.

Note: For Si the electron-effective mass is $m_e^* = 0.33\ m_e$, and the relative permittivity is $\varepsilon_r = 11.7$. The Bohr radius for a hydrogen atom is $r_n = (n^2\ \hbar^2/m_e\ e^2)\ 4\pi\ \varepsilon_0$, where the symbols have their usual meaning, and has the value 0.53 Å for $n = 1$.

[7 marks]

1.7) The rectifier equation for an abrupt p-n junction is $I = I_0\ [\exp(eV/k_B T) - 1]$, where I is the forward current at bias voltage V and temperature T.

Sketch this function, illustrating the significance of the term I_0, and explain why a real junction rapidly departs from this behaviour at large values of V.

[7 marks]

1.8) Explain what is meant by the ‘Meissner effect’ for a type I superconductor. Show that such a material behaves as a perfect diamagnet.

[7 marks]

SECTION B - answer two questions

2.) In a one-dimensional chain, atoms of masses M and m alternate, and are coupled together by springs with a force-constant K. Atoms of the same type are separated by a distance a.

The relative phase and amplitude of the vibrations of the atoms are described by a complex number α, and, assuming only nearest-neighbour forces are important, it is straightforward to show that:

$$\alpha = [2K \cos(\tfrac{1}{2}ka) / (2K - \omega^2 m)] = [(2K - \omega^2 M) / 2K \cos(\tfrac{1}{2}ka)]$$

where ω is the angular frequency and k is the wavenumber.

Hence show that

$$\omega^2 = [K(M+m)/Mm]\,\{1 \pm [1 - [4Mm/(M+m)^2] \sin^2(\tfrac{1}{2}ka)]^{1/2}\},$$

[8 marks]

Show that, as $k \to 0$, there is a high-frequency mode where the atoms with different masses oscillate in antiphase, and a low-frequency mode where the atoms oscillate with the same amplitude and phase.

[7 marks]

Sketch the remaining relationships between ω and k, and explain why the upper branch is called the 'optical branch' and the lower branch is called the 'acoustic branch.'

[5 marks]

Sodium chloride exhibits intense absorption at a wavelength of 61 μm in the infrared spectral region. Assuming that the formulae derived for a linear chain apply approximately to a three-dimensional crystal, estimate the force constant for the ionic bond. The atomic mass numbers of Na and Cl are 23 and 35.5, respectively.

[10 marks]

See next page

3.) Explain what is meant by an 'indirect gap semiconductor' and describe the 'phonon absorption' and 'phonon emission' processes by which an electron may be optically excited across the indirect gap.

[12 marks]

The absorption coefficient in an indirect gap semiconductor for a photon energy $h\nu$ is proportional to $(h\nu - h\nu_{min})^2$, where $h\nu_{min}$ is the minimum energy to cross the gap.

Describe, with appropriate analysis, how absorption spectra may be plotted to estimate the energy gap and principal phonon energy. Indicate also how the spectra vary with temperature.

[12 marks]

Discuss why an indirect gap semiconductor is unlikely to be suitable for the fabrication of diode lasers.

[6 marks]

4.) Krönig and Penney showed that electrons moving in a periodic potential can take energies E related to the wavenumber k by

$P\,[\sin(\alpha a)]/\alpha a + \cos(\alpha a) = \cos(ka)$, where

$P = m_e a V_0 b/\hbar^2$, and

$\alpha = \sqrt{(2m_e E)}/\hbar$.

The period of the lattice is $(a + b)$ and the height of the potential barrier is V_0. The product $V_0 b$ is constant, and $b << a$.

Using a diagram, show how this relationship leads to a situation in which allowed energy bands are separated by forbidden energy bands. Show further that when $V_0 = 0$ (as in a metal) the solution reduces to the free-electron parabola $E = \hbar^2 k^2/2m_e$.

[15 marks]

Draw a diagram, using the repeated zone scheme, to illustrate how the Krönig and Penney solutions are related to the free-electron parabola. By considering the discontinuities that occur on such a diagram, explain how the energy gaps may be interpreted in terms of Bragg diffraction of the electron waves.

[15 marks]

See next page

5.) Show that, when the electron and hole concentrations, n and p respectively, in a semiconductor are similar, the Hall coefficient is given by
$R_H = (p - b^2 n)/e(p + nb)^2$,
where $b = \mu_e/\mu_h$ is the ratio of the electron mobility to the hole mobility.

You may assume that, in an electric field **E** and a magnetic flux density **B**, the electron and hole velocities are given, respectively, by
$\mathbf{v}_e = -\mu_e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B})$, and
$\mathbf{v}_h = \mu_h (\mathbf{E} + \mathbf{v}_h \times \mathbf{B})$.

[12 marks]

Use the equation for R_H, above, to derive an expression for the Hall coefficient of an intrinsic semiconductor, and hence explain why, for most semiconductors, when the temperature is raised sufficiently, the Hall coefficient changes sign for p-type material, but not for n-type material.

[8 marks]

The Hall coefficient is zero for a certain sample of p-type germanium at 75 °C. From this observation, calculate the acceptor concentration N_a. For intrinsic Ge the hole concentration is $p_i = 1.5 \times 10^{21} T^{3/2} \exp(-E_g/2k_B T)$ at temperature T, and at 75 °C the energy gap E_g is 0.646 eV and $b = 2$.

You may assume that $p = N_a + n$ and that $np = p_i^2$.

[10 marks]