King’s College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/3402 Solid State Physics

Summer 2002

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

<u>Fundamental constants</u>

Boltzmann constant k_B = 1.381×10^{-23} J K^{-1} = 8.617×10^{-5} eV K^{-1}
Avogadro number N_A = 6.022×10^{23} mol^{-1}

The following symbols are used throughout the paper, but their values are not required.

Reduced Planck constant $\hbar$
Elementary charge e
Rest mass of an electron m_e
Permittivity of free space e_0

SECTION A - Answer SIX parts of this section

1.1) Describe the face centred cubic structure. Show that, if the atoms are regarded as hard spheres in close contact, 26 % of the structure is empty space.

[7 marks]

1.2) The two components of the K_α radiation from copper have wavelengths of $K_{\alpha 1}$ = 0.15405 nm and $K_{\alpha 2}$ = 0.15443 nm. For a certain crystal the angle of diffraction for $K_{\alpha 2}$ is 88 °. Calculate the angle of diffraction for $K_{\alpha 1}$.

[7 marks]

1.3) The relation between angular frequency w and wavenumber k for waves propagating on a linear chain of atoms of mass M spaced a distance a apart is $w^2 M = 4K \sin^2(\tfrac{1}{2} ka)$, where K is the "spring constant."

Sketch the dispersion curve of w versus k. Show that an extrapolation, using the velocity of sound waves to estimate the maximum frequency of vibration, overestimates the true value by a factor of $\pi/2$.

[7 marks]

1.4) The heat capacity C_v of a metallic crystal at low temperature T may be written as $C_v = \gamma T + bT^3$.

What is the origin of these two contributions to C_v? How may low-temperature experimental data be plotted in terms of T to obtain a straight line graph?

[7 marks]

1.5) The resistivity r for a perfect metal crystal with an electron concentration n is $r = \frac{m_e}{ne^2 t}$ where t is the relaxation time. Show how this expression may be modified for a slightly imperfect crystal.

[7 marks]

1.6) Explain how elements from group III of the periodic table form substitutional acceptors in group IV elements. Estimate the ionisation energy of the boron acceptor in diamond using the hydrogen model analogy, given that the ionisation energy of the hydrogen atom is

$$E_1 = -\frac{m_e e^4}{2\hbar^2 (4pe_0)^2} = 13.6 \text{ eV}.$$

[For diamond the effective mass of the holes is $m_h^* = 0.75\ m_e$, and the relative permittivity is $e_r = 5.70$.]

[7 marks]

1.7) Explain what is meant by an *indirect gap* semiconductor. Describe the two processes by which electrons may be excited optically from the top of the valence band to the bottom of the conduction band in such a material.

[7 marks]

1.8) Sketch how the carrier concentration changes with temperature in a partially compensated semiconductor, drawing attention to the intrinsic region, the saturated extrinsic region and the carrier freeze-out region. Which energies can be measured from the gradients in the different regions?

[7 marks]

SECTION B - Answer TWO questions

2) Explain the meanings of the terms in the expression for the structure factor F_{hkl} in relation to the diffraction of X-rays from a crystalline solid:

$$F_{hkl} = \sum_j f_j \exp\{2\pi i(h x_j + k y_j + l z_j)\}.$$

[3 marks]

For a given angle of diffraction, on what does the parameter f_j principally depend?

[2 marks]

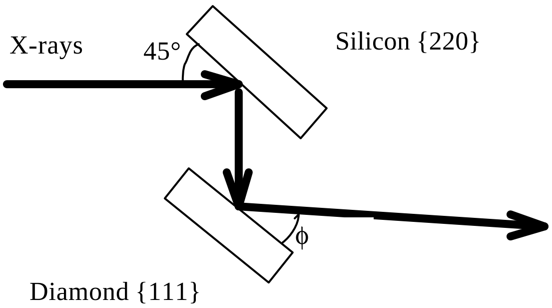
Write down the structure factor for the diamond crystal structure and hence show that space group extinctions occur unless h, k, l are all odd or all even with $h + k + l = 4m$ where m is an integer.

[15 marks]

In the diagram below, “white” X-rays from a synchrotron are diffracted by a silicon crystal inclined at 45° to the beam. X-rays produced from the {220} reflection are subsequently used to investigate a diamond. Calculate the angle ϕ at which the diamond must be oriented with respect to the diffracted beam in order to observe the {111} reflection from diamond.

[The lattice constants of silicon and diamond are 0.543 and 0.3567 nm, respectively.]

[10 marks]



3) State the assumptions made by Einstein to obtain an expression for the temperature-dependence of the heat capacity of a crystalline solid.

[3 marks]

At temperature *T*, the average energy of a quantum oscillator with angular frequency ω is given by

$$\overline{E} = \frac{\hbar w}{2} + \frac{\hbar w}{\left[\exp(\hbar w / k_B T) - 1\right]} \quad .$$

(a) Show that the Einstein expression for the heat capacity per mole may be written as

$$C = 3N_A k_B \left(\frac{q_E}{T} \right)^2 \frac{\exp(q_E / T)}{\left[\exp(q_E / T) - 1\right]^2} \ ,$$

where q_E is the Einstein temperature, w_E is the Einstein frequency and $q_E = \hbar w_E / k_B$.

[8 marks]

(b) Discuss the behaviour of this expression at (i) high temperature and (ii) low temperature.

[5 marks]

(c) Discuss how the behaviour predicted from the Einstein expression compares with that observed experimentally. Explain, without derivation, the more realistic approach used by Debye to predict the temperature dependence of the heat capacity.

[4 marks]

(d) The variation of heat capacity with temperature for silicon can be fitted reasonably well using q_E = 380 K. Show that approximately 125 kJ of energy is required to cool 1 kg of silicon from room temperature (293 K) to liquid nitrogen temperature (77 K).

[The atomic mass number of silicon is 28.]

[10 marks]

4) Explain what is meant by the Hall effect, and define the Hall coefficient.

[4 marks]

Derive an expression for the Hall coefficient of a metal with an electron concentration of n.

[4 marks]

In a semiconductor, when the electron and hole concentrations, n and p respectively, are similar, the Hall coefficient is given by

$$R_H = \frac{p - b^2 n}{e(p + nb)^2}$$

where $b = \mu_e/\mu_h$ is the ratio of the electron mobility to the hole mobility.

Show that this result is consistent with the expression derived for a metal.

[2 marks]

Use the equation for R_H, above, to derive an expression for the Hall coefficient of an intrinsic semiconductor, and hence explain why, for most semiconductors, when the temperature is raised sufficiently, the Hall coefficient changes sign for p-type material, but not for n-type material.

[8 marks]

The Hall coefficient is zero for a certain sample of p-type germanium at 75 °C. From this observation, calculate the acceptor concentration N_a.

[For intrinsic Ge the hole concentration is $p_i = 1.5 \text{ x } 10^{21}\ T^{3/2} \exp(-E_g/2k_BT)$ at temperature T, and at 75 °C the energy gap E_g is 0.646 eV and $b = 2$.]

[You may assume that $p = N_a + n$ and that $np = p_i^2$.]

[12 marks]

5) Explain:

(i) what is meant by an abrupt p-n junction;

(ii) why there is a potential difference built up between the n and p regions;

(iii) what is meant by a depletion layer and why it forms;

(iv) why a p-n junction possesses capacitance (qualitative explanation, only).

[12 marks]

Show that the current I flowing through a p-n junction at a bias voltage V is $I = I_0\,[\exp(eV/k_BT) - 1]$, where I_0 is the reverse saturation current.

[10 marks]

At 293 K a certain silicon p-n junction rectifier has a reverse saturation current of 10^{-10} A, and the combined resistance of the material on either side of the junction is 0.1 Ω. Calculate the voltage dropped across the whole device for a forward current of 4 A.

[At 293 K, $e/k_BT = 40\ \mathrm{V}^{-1}$.]

[8 marks]

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