King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3402 Solid state physics

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Fundamental constants

Planck constant	h	$= 6.626 \times 10^{-34}$ J s
Elementary charge	e	$= 1.602 \times 10^{-19}$ C
Rest mass of an electron	m_e	$= 9.109 \times 10^{-31}$ kg
Permittivity of free space	ε_0	$= 8.854 \times 10^{-12}$ F m^{-1}
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}$ H m^{-1}
Boltzmann constant	k_B	$= 1.381 \times 10^{-23}$ J K^{-1} $= 8.617 \times 10^{-5}$ eV K^{-1}
Avogadro number	N_A	$= 6.022 \times 10^{23}$ mol^{-1}

SECTION A – Answer SIX parts of this section

1.1) Assuming that the atoms may be regarded as hard solid spheres in close contact, show that almost $^2/_3$ of a silicon crystal is empty space.

[7 marks]

1.2) Explain how the Miller indices (hkl) are calculated for a crystal plane.

Diamond grown by high-pressure, high-temperature synthesis can exhibit crystal faces with orientations of {100}, {110}, {111} and, occasionally {113}. For each of these orientations sketch a unit cube illustrating the respective plane.

[7 marks]

1.3) Show that, for long wavelengths, the velocity of longitudinal waves in a crystal is $v_L = (C/\rho)^{1/2}$ where C is an appropriate elastic modulus and ρ is the density.

[7 marks]

1.4) Explain why the contribution of electrons to the heat capacity of a metal is negligible at room temperature, but becomes dominant as the temperature approaches zero.

[7 marks]

1.5) Give a brief description of three different processes by which optical radiation may be absorbed by a semiconducting or insulating crystalline solid.

[7 marks]

1.6) The Hall coefficient for a semiconductor with electron and hole concentrations n and p, respectively, is given by $R_H = \dfrac{p - b^2 n}{e(p + nb)^2}$ where $b = \mu_e/\mu_h$ is the ratio of the electron mobility to the hole mobility.

Explain why, for most semiconductors, the sign of R_H reverses at high temperatures for p-type material, but not for n-type material.

[7 marks]

1.7) The rectifier equation for an abrupt p-n junction is $I = I_0\ [\exp(eV/k_B T) - 1]$, where I is the forward current at bias voltage V and temperature T.

Sketch this function, illustrating the significance of the term I_0, and explain why a real junction rapidly departs from this behaviour at large values of V.

[7 marks]

1.8) Explain, briefly, the Silsbee hypothesis.

The type I superconductor tin has a critical temperature T_c = 3.7 K, and a critical field B_c = 30.6 mT at temperature T = 0. Calculate the critical current for a tin wire of radius 1 mm at T = 2 K.

[7 marks]

SECTION B – Answer TWO questions

2) Explain the meanings of the terms in the expression for the structure factor F_{hkl} in relation to the diffraction of X-rays from a crystalline solid:

$$F_{hkl} = \sum_j f_j \exp\{2\pi i\,(h\,x_j + k\,y_j + l\,z_j)\}.$$

[3 marks]

For a given angle of diffraction, on what does the parameter f_j principally depend?

[2 marks]

Use this information to derive the *extinction rules* for the sodium chloride crystal structure.

[8 marks]

$^{23}_{11}$Na and $^{19}_{9}$F form the alkali halide NaF which crystallises in the sodium chloride structure, and has a density of 2780 kg m^{-3}.

(i) Stating any assumptions you make, show that the lattice constant of the cubic unit cell can be estimated as a = 0.465 nm.

[5 marks]

(ii) Calculate the smallest and largest angles of diffraction when a polycrystalline sample of NaF is placed in a collimated beam of X-rays with wavelength 0.2 nm.

[8 marks]

(iii) If, in this example, the $^{23}_{11}$Na were replaced by $^{5}_{3}$Li, calculate the ratio of the intensities of the strong and weak reflections.

[4 marks]

3) Explain what is meant by the Fermi energy at 0 K for a metal crystal.

[2 marks]

Starting from the Schrödinger equation, show that the Fermi energy for N free electrons in a metal crystal of volume V is given by

$$E_F = \frac{h^2}{8m_e}\left(\frac{3N}{\pi V}\right)^{2/3}$$

[13 marks]

The concentration of free electrons in aluminium is $1.807 \times 10^{29}\ \text{m}^{-3}$.

(i) Calculate the Fermi velocity of these electrons.

[5 marks]

(ii) An aluminium wire of radius 1 mm carries a current of 10 amps.

(a) Calculate the drift velocity of the electrons, and compare this velocity with the Fermi velocity.

[5 marks]

(b) The potential drop in the wire is 84 mV m^{-1}. Calculate the electron mobility.

[5 marks]

4) By considering the effect of an electric field on a wavepacket with energy E and wavenumber k, show that the effective mass m^* of an electron moving in a periodic potential is

$$m^* = \hbar^2 \bigg/ \frac{d^2E}{dk^2}$$

[12 marks]

Describe, paying attention to the experimental conditions necessary, how effective masses in a semiconductor can be measured using cyclotron resonance. Show that resonance is obtained at a frequency $f = eB / 2\pi m^*$ for a magnetic flux density B.

[10 marks]

Sketch the E-k curves for silicon. For the valence band identify the light hole band, the heavy hole band and the split-off band.

[3 marks]

Measurements on silicon, using a microwave cavity operating at 24 GHz, yielded resonances at B = 0.185 and 0.327 T for electrons, and 0.139 and 0.437 T for holes. Calculate the effective masses of the light and heavy electrons, and the light and heavy holes, expressing the results as multiples of the free electron mass.

[5 marks]

5) Explain how elements from group V of the periodic table form substitutional donors in silicon and germanium. Estimate the ionisation energy of such a donor using the *hydrogen model analogy*, given that the ionisation energy of the hydrogen atom is

$$E_1 = -\frac{m_e e^4}{2\hbar^2 (4\pi\varepsilon_0)^2} = -13.6 \text{ eV}$$

where ε_0 is the permittivity of free space.

For Si the effective mass of the electrons is $m_e^* = 0.33\, m_e$, and the relative permittivity is $\varepsilon_r = 11.7$.

[7 marks]

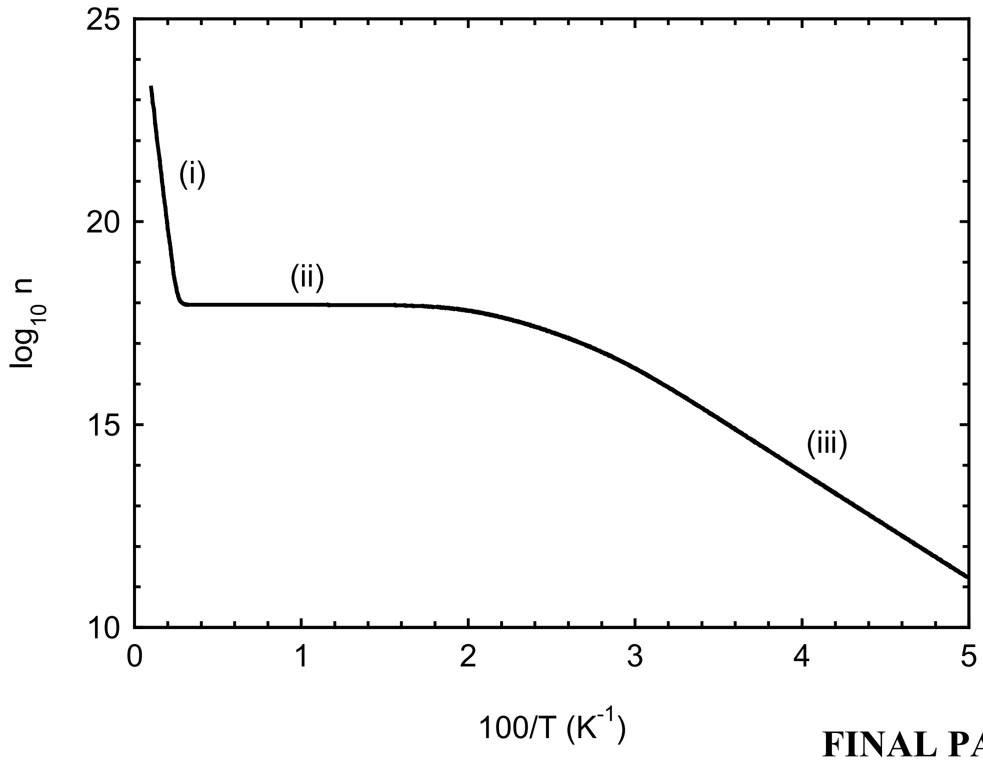
The diagram below shows, for a partially compensated n-type semiconductor, the variation with inverse temperature of $\log_{10}(n)$, where n is the electron concentration per m^3.

Explain, without mathematical derivation, why the curve has this shape, paying particular attention to regions (i), (ii) and (iii).

[8 marks]

By making measurements from the diagram, estimate the energy gap of the semiconductor, the effective donor concentration and the donor ionisation energy.

[15 marks]



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