King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP3380 Optics

Examiner: Dr. G.R. Morrison

Summer 2007

Time allowed: THREE Hours

Candidates should answer ALL parts of SECTION A,
and no more than TWO questions from SECTION B.

No credit will be given for answering a further question from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You may only use a College-approved calculator for this paper.

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

SECTION A – Answer ALL parts of this section

1.1) A transmission diffraction grating is illuminated by light from a sodium vapour lamp. How many slits of the grating must be illuminated if the first-order principal maxima of the two yellow sodium D lines (with wavelengths 588.995 nm and 589.592 nm) are to be just resolved?

If the grating has 400 slits per mm, what is the angular separation between the *third-order* principal maxima produced by the two sodium D lines?

[6 marks]

1.2) Explain briefly what is meant by a *weak phase object*.

Describe how the Schlieren method can be used to produce an image of a weak phase object.

[7 marks]

1.3) Explain briefly how a *Pupil Function* is used to characterise the performance of a simple lens-based imaging system that uses coherent illumination.

State the mathematical relationship between the *Pupil Function* and the *Optical Transfer Function* for the same system when using incoherent illumination.

[7 marks]

1.4) Explain briefly whether the following statements are true or false.

a) The resolution of an imaging system could be improved by using illumination that has a shorter wavelength.

b) The resolution of an imaging system could be improved by adding an additional lens to the final part of the optical system, to produce an image with larger magnification.

c) The resolution of a holographic image recording could be improved by using a film emulsion with a smaller grain size.

d) A laser source is essential for the successful recording of a holographic image.

[9 marks]

1.5) Identify three advantages of the off-axis geometry for recording a Fresnel hologram when compared to the in-line (or Gabor) geometry.

[5 marks]

1.6) A reflection hologram was recorded using yellow light of wavelength 600 nm from a laser source. During processing of the exposed hologram, the linear dimensions of the photographic emulsion were caused to shrink by about 10%, so that the spacing of the interference fringes that make up the hologram was changed. Explain the effect this might have on the appearance of the holographic image when the hologram is illuminated by a beam of white light from a small spot-lamp some distance from the hologram.

[6 marks]

SECTION B – Answer TWO questions

2a) A one-dimensional slit aperture has transmission function $f(x)$. Write down an expression for the amplitude in the far-field diffraction pattern produced when this aperture is illuminated by a monochromatic plane wave propagating along a normal to the aperture.

[2 marks]

b) A diffraction grating consists of a regular linear array of N identical apertures, with the apertures separated by a centre-to-centre distance d. *Without giving a detailed derivation,* outline how to obtain an expression for the diffracted intensity from the grating that is of the form

$$I(u) = I_0 \, |F(u)|^2 \left(\frac{\sin N\pi ud}{\sin \pi ud} \right)^2$$

taking care to define each of the terms in the equation above. Hence derive an expression for the values of u at which the principal maxima will occur in the far-field diffraction pattern.

[10 marks]

c) A particular design of diffraction grating has apertures of width $a = d$, so that the grating has no opaque regions, but each aperture incorporates a phase-shifting filter that produces a transmission function of the form $f(x) = \exp(2\pi i\beta x)$, for $-a/2 \leq x < a/2$. Show that, in this case,

$$|F(u)|^2 = a^2 \left[\frac{\sin \pi(u-\beta)a}{\pi(u-\beta)a} \right]^2$$

[8 marks]

Hence show that, when $\beta = 5/a$, the intensities of all but one of the principal maxima in the diffraction pattern will be zero. Determine the value of u at which the one principal maximum with non-zero intensity occurs.

Briefly suggest why a grating with these characteristics might be useful.

[10 marks]

3) The optical forms of the amplitude transmission coefficients for p and s polarised light are

$$T_p = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1} \qquad \text{and} \qquad T_s = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_2 + n_1 \cos\theta_1}$$

respectively, where subscripts p and s have their usual meanings, θ_1 represents the angle of incidence in a medium of refractive index n_1, and θ_2 represents the angle of refraction in a medium of refractive index n_2.

a) Light incident on the boundary between the two media is linearly polarised at an angle α to the plane of incidence. Explain carefully why the direction of polarisation of the light transmitted into medium 2 is generally different from that of the incident light.

[4 marks]

b) Assuming that the polarisation angle for the incident light is $\alpha = 60°$, determine the direction of polarisation of the transmitted light when $\theta_1 = 50°$, $n_1 = 1.3$, $n_2 = 1.5$.

[8 marks]

c) Starting with either the expression for T_p or T_s given above, derive an expression for the intensity reflectance $\mathcal{R}$ at normal incidence to the boundary between the two media.

[6 marks]

d) Consider the case where light is incident from medium 1 into medium 2, which is a plane sheet of material that covers a third medium of refractive index n_3, where $n_3 \neq n_1$. Show that when $n_2 = \sqrt{n_1 n_3}$ the fraction of the light intensity reflected at each boundary of medium 2 is the same.

[7 marks]

e) Determine suitable values for the thickness of medium 2 and the refractive index n_3 that will minimise the overall fraction of incident light reflected back into medium 1, assuming the light has a wavelength 720 nm.

[5 marks]

4a) Laser operation requires that the rate of stimulated emission is greater than the rate of resonance absorption. Show that this requires a *population inversion.*

[5 marks]

b) Outline briefly how a population inversion can be achieved with a 3-level laser system. Comment on the advantages of using a 4-level system instead.

[10 marks]

c) If the output from a laser is required to be linearly polarised, explain how Brewster windows could be used to achieve this, and suggest one reason why this method is preferable to the use of a simple polarising filter on the light emitted by the laser.

[6 marks]

d) A gas laser system with an 80 cm long resonant cavity has laser emission at a wavelength of 543 nm, with a Doppler-broadened transition width $\Delta\nu \approx$ 1.5 GHz. Assuming the gas refractive index $n = 1$, determine the number of resonant longitudinal modes that can be sustained by this laser and calculate the maximum cavity length that would ensure single-mode operation of the laser.

[6 marks]

e) If the linewidth of a single resonant mode is 2.5 MHz, estimate the coherence time and longitudinal coherence length for the laser output when it is operated in single mode form.

[3 marks]