King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP3380 Optics

Summer 2006

Time allowed: THREE Hours

Candidates should answer ALL parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_{e}	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_{u}	=	1.661×10^{-27}	kg
Proton rest mass	m_{p}	=	1.673×10^{-27}	kg
Neutron rest mass	m_{n}	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_{B}	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_{A}	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

SECTION A – Answer ALL parts of this section

1.1) With the aid of a diagram, explain how the addition of a thin transparent film to the surface of a glass lens can significantly increase the fraction of light transmitted through the lens.

The lens is made of glass with a refractive index of 1.69, and is to be used mainly with yellow light of wavelength 590 nm. Discuss how this influences the specifications of the thin film coating the lens.

[8 marks]

1.2) A plane wave is incident on the boundary between two transparent media of refractive indices n_1 and n_2 respectively. For light polarised *parallel* to the plane of incidence, the amplitude reflection coefficient R_p for light at an incident angle θ_1 in medium 1 is given by

$$R_p = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

where θ_2 is the angle of refraction in medium 2. Derive an expression for the intensity reflectance for light incident normally on to the boundary between the two transparent dielectric media, and hence show that the intensity transmittance at normal incidence is given by

$$\mathcal{T} = \frac{4n_2 n_1}{(n_1 + n_2)^2}$$

[6 marks]

1.3) Explain briefly whether the following statements are true or false:

a) The angular separation of the principal maxima in the Fraunhofer diffraction pattern from a linear diffraction grating will increase as the wavelength of the illumination is increased.

b) The angular separation of the principal maxima in the Fraunhofer diffraction pattern from a linear diffraction grating will increase as the period of the grating is increased.

c) The chromatic resolving power of a linear diffraction grating will increase as the illuminated width of the grating is increased.

[6 marks]

1.4) A child with good vision observes two point-like objects in daylight, at a distance of 40 cm from her face. Assuming the mean wavelength of daylight is 600 nm, and that the pupils of the child's eyes are 3 mm in diameter, estimate the smallest separation of the two objects that could be resolved by the child.

[6 marks]

1.5) Distinguish between the recording conditions needed to produce an *in-line (or Gabor) Fresnel hologram* and a *Fourier-transform hologram* of a thin transmitting object.

Explain why electronic detectors, such as CCD cameras, can be used more easily in the case of the Fourier-transform geometry than for the in-line Gabor method.

[8 marks]

1.6) A Fresnel hologram is to be recorded with coherent monochromatic light of wavelength 450 nm using a sheet of photographic emulsion that is capable of resolving 600 lines per mm and lies perpendicular to the direction of the reference wave. Calculate the maximum possible angle between the reference wave and any scattered waves that can contribute to the formation of the hologram.

If the minimum resolvable features in the holographic image are 2 μm in size, calculate the numerical aperture of the lens that would be required to produce a photographic image of the same object with comparable resolution, assuming the same coherent illumination was used.

[6 marks]

SECTION B – Answer TWO questions

2a) Briefly distinguish between the conditions needed to observe Fraunhofer and Fresnel diffraction.

A small iris diaphragm is illuminated by monochromatic light and a diffraction pattern is observed some distance from the aperture. Suggest two ways in which it would be possible, *without making any measurements of distances or aperture sizes*, to determine whether the observed diffraction pattern is more likely to be Fresnel-like or Fraunhofer-like in character.

[8 marks]

b) A particular object is illuminated with a monochromatic plane wave of wavelength λ. The diffracted intensity in the far field is found to have the form

$$I(u,v) = I_0 \left(\frac{J_1\left(\pi a\sqrt{u^2+v^2}\right)}{\pi a\sqrt{u^2+v^2}} \right)^2 \left(\frac{\sin 3\pi d_1 u}{\sin \pi d_1 u} \right)^2 \left(\frac{\sin 5\pi d_2 v}{\sin \pi d_2 v} \right)^2$$

where $J_1()$ represents a first-order Bessel function of the first kind.
Briefly comment on the physical significance of each of the bracketed terms in the equation. Without carrying out a detailed calculation, explain clearly the deductions that can be made about the general form of the diffracting object.

[12 marks]

c) A linear diffraction grating, with 1500 rectangular slits, is illuminated by red light from a lithium source. This light consists of two closely spaced wavelengths with a mean wavelength of 670.8 nm. A series of bright diffraction spots is observed in the diffraction pattern 1.25 m from the grating. The diffraction spots near the centre of the pattern are evenly spaced with a mean separation of 10 mm in the diffraction plane, except that on moving away from the centre of the pattern every 7th spot is absent. For the 30th-order and higher-order diffraction spots (taking zero at the centre of the pattern) it is found that each spot can be resolved into two spots very close together.

Determine the values of the slit width and the grating period, and calculate the difference in the wavelengths of the two red lithium lines.

[10 marks]

3a) State briefly the two principal ideas that form the basis of Abbe's theory of image formation with a lens. With the aid of a diagram, show how these ideas can be applied to the case of a simple optical system that consists of two identical biconvex lenses of focal length $f = 20\,\text{cm}$ and diameter $D = 5\,\text{cm}$ that are placed coaxially a distance $2f$ apart.

A circular aperture of variable diameter D_a (where $D_a < D$) is centred on the optical axis and located in the common focal plane midway between the two lenses. Comment on the effect on the image of reducing the aperture diameter D_a.

[9 marks]

b) An object is placed in the front focal plane of the first lens. It has a transmission function of the form

$$f(x) = [1 + a\cos(2\pi x/d)]$$

where $0 < a \leq 1$, d is a constant length and x represents a distance perpendicular to the optical axis.

Assuming this object is illuminated with coherent illumination of wavelength 500 nm, show that there will be three diffraction spots produced in the back focal plane of the first lens.

Calculate the minimum diameter D_a that would allow these spots to pass through, assuming $d = 20\,\mu\text{m}$.

[10 marks]

c) The object is now replaced with another having a transmission function

$$h(x) = f(x)g(x)$$

where $f(x)$ is as given above, and $g(x)$ is an unknown function with Fourier transform $G(u)$. This means that there are cosine-like fringes superimposed on the unknown object structure of interest.

Use the convolution theorem for Fourier transforms to show that the complex amplitude in the back focal plane of the first lens is now given by

$$E_0\left[G(u) + \frac{a}{2}\left[G(u - 1/d) + G(u + 1/d)\right]\right]$$

where E_0 is a constant.

[6 marks]

d) Suggest how the size of the variable aperture D_a could be adjusted so that the cosine modulation is removed from the spatially-filtered image intensity, which is then proportional to $|g(x)|^2$.

[5 marks]

4a) Write down expressions that describe the rates of (i) spontaneous emission, (ii) resonant absorption and (iii) stimulated emission in a two-level atomic system, defining all the symbols used.

[4 marks]

b) With the aid of a diagram, explain how a population inversion is achieved in a four-level laser.

[6 marks]

c) Describe three benefits that the use of a resonant cavity brings to the operation of a laser. List three effects that can lead to a loss of light intensity within the resonant cavity of a working laser.

[9 marks]

d) If the output from a laser is required to be linearly polarised, explain how Brewster windows could be used to achieve this, and suggest one reason why this method is preferable to the use of a simple polarising filter on the light emitted by the laser.

[6 marks]

e) A semi-conductor laser has a resonant cavity that is 0.24 mm long, and the laser emits infra-red light. The refractive index of the semiconducting material at the wavelength of the lasing transition is 2.6. Calculate the frequency separation of successive resonant longitudinal modes that could be sustained by the laser.

[5 marks]