King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP3380 Optics

Summer 2005

Time allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

SECTION A – Answer all SIX parts of this section

1.1) A diamond film is coated on the surface of a glass window. If the refractive index of diamond is $n_1 = 2.42$, and the refractive index of the glass window is $n_2 = 1.50$, calculate the critical angle for total internal reflection when light is incident on the boundary between the diamond film and the window.

Draw a clearly-labelled diagram showing the ray paths when a parallel beam of light is incident on the boundary at the critical angle.

[7 marks]

1.2) With the aid of a diagram, distinguish between the *spatial* and *temporal* coherence of a beam of light.

State how the temporal coherence *length* can be related to the coherence *time.*

[7 marks]

1.3) *Polaroid*™ filters have the property that they transmit light of only one direction of polarisation. Explain briefly how *Polaroid* sunglasses can be used to reduce the "glare" of sunlight reflected from the surface of a swimming pool, assuming the refractive index of water is 1.33.

[7 marks]

1.4) State what is meant by *phase contrast imaging.*

Identify two important differences between the Schlieren and the Zernike methods of phase contrast imaging.

[7 marks]

1.5) State the main principle involved in the holographic recording of an image.

Identify two advantages of the off-axis geometry for recording a Fresnel hologram when compared to the in-line (or Gabor) geometry.

[7 marks]

1.6) With the aid of a diagram, explain how it is possible to use white light to view the holographic image produced by a reflection hologram, even though monochromatic illumination is needed to record the hologram.

[7 marks]

SECTION B – Answer TWO questions

2a) Discuss briefly how the convolution theorem for Fourier transforms can be used when dealing with the problem of Fraunhofer diffraction from arrays of identical apertures. Hence explain the presence of two terms in the expression for the far-field diffracted intensity, known respectively as the *Diffraction Envelope* and the *Interference Term*.

[10 marks]

b) Figure 2.i shows five objects from which the Fraunhofer diffraction pattern is to be obtained. Figure 2.ii shows five possible Fraunhofer diffraction patterns. Identify which of the diffraction patterns should be associated with each of the diffracting objects shown, briefly explaining the reasons for each of your choices.

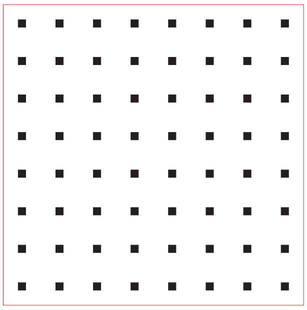
[10 marks]

c) Derive an expression for the far-field diffraction pattern obtained from a set of N identical apertures that are positioned randomly in a plane, assuming that N is a very large number, and that none of the apertures overlap.

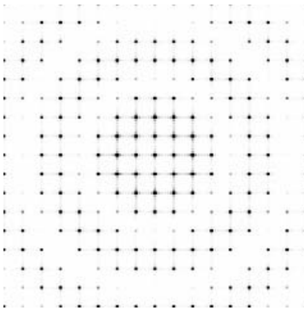
[7 marks]

d) If a very large number of circular apertures, the same size as those shown in Object 4 of Figure 2.i, are positioned randomly in the object plane, without any of them overlapping, describe qualitatively the form of the Fraunhofer diffraction pattern that you would expect to observe.

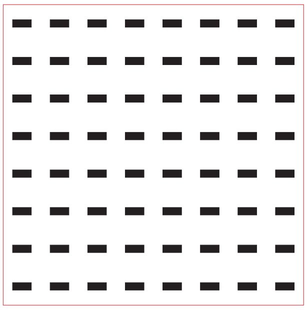
[3 marks]



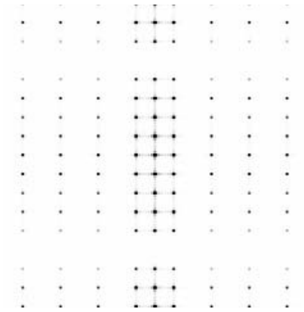
2.i Object 1



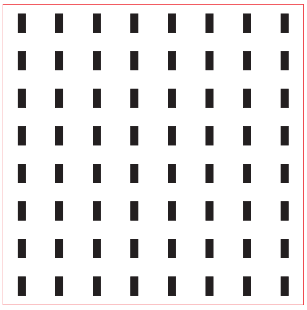
2.ii Diffraction pattern A



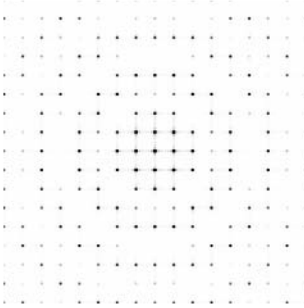
2.i Object 2



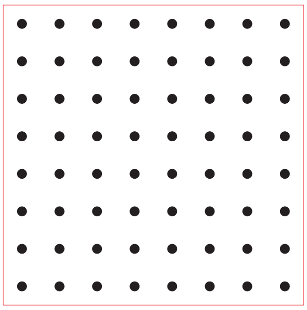
2.ii Diffraction pattern B



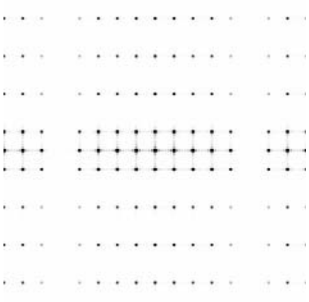
2.i Object 3



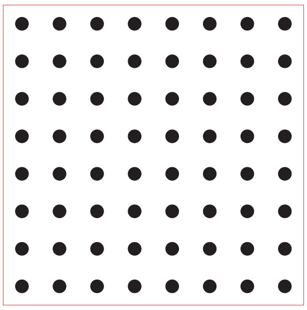
2.ii Diffraction pattern C



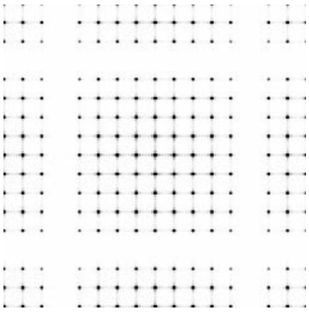
2.i Object 4



2.ii Diffraction pattern D



2.i Object 5



2.ii Diffraction pattern E

3a) State briefly the two principal ideas that form the basis of the Abbe theory of image formation, and explain why the concept of a *pupil function* is useful when analysing the behaviour of a lens-based imaging system.

State the mathematical relationship between the *pupil function* of an optical system and the *point spread function* for the same system.

[10 marks]

b) An object with transmission function $f(x)$ is placed in the front focal plane of a simple two-lens imaging system, and is imaged with coherent illumination of wavelength 600 nm. The two lenses are identical, with apertures of diameter 20 mm and focal lengths of 50 mm, and they are separated by an axial distance of 100 mm. The medium between the object and the first lens has refractive index $n = 1$.

(i) Calculate the effective numerical aperture of the optical system, assuming there is no mask or aperture in the back focal plane of the first lens.

[4 marks]

(ii) The complex amplitude in the back focal plane of the first lens is found to be of the form

$$E_0 \left[\frac{1}{2}\delta(u) + \frac{A}{12}\left[\delta(u-k) + \delta(u+k)\right] + \frac{A}{20}\left[\delta(u-4k) + \delta(u+4k)\right] \right]$$

where $\delta()$ is a Dirac delta function, and A is a positive real constant, with $A \ll 1$. A circular aperture is placed in the back focal plane of the first lens. In the u direction, the pupil function can be assumed to be of the form

$$P(u) = \begin{cases} 1 & \text{if } |u| \leq 2k, \\ 0 & \text{elsewhere.} \end{cases}$$

Show that, in this case, the intensity in the image plane formed by the second lens is given by the expression

$$I(x) = \frac{|E_0|^2}{4} \left[1 + \frac{A}{3} \cos\left(2\pi k x\right) \right]^2 \tag{3.1}$$

Show also that the *amplitude* contrast in the image plane is

$$C = \frac{A}{3}$$

[10 marks]

(iii) Given that $k = 2 \times 10^4\,\mathrm{m}^{-1}$, calculate the diameter of the aperture used as a mask in part (ii). What is the minimum value to which this aperture diameter could be reduced without affecting the image intensity distribution given by equation (3.1)?

[6 marks]

4a) Laser operation requires that the rate of stimulated emission exceeds the rate of resonant absorption. Show that this requires the existence of a *population inversion* within the lasing medium. Briefly explain why it is not generally practicable to sustain continuous light emission from a 3-level laser.

[9 marks]

b) A photon produced by stimulated emission has the same phase, polarisation, frequency and direction of travel as the stimulating photon. Explain why these properties are not sufficient to ensure that a laser produces a collimated beam of coherent monochromatic light, and describe how the use of a resonant cavity can help to produce this form of output.

[8 marks]

c) Show that for a resonant cavity of length L the frequency difference between consecutive longitudinal resonant modes is $\Delta\nu = c/2nL$, where c is the free-space velocity of light, and n is the refractive index of the medium within the cavity.

[3 marks]

d) For a particular laser, it is found that stimulated emission occurs simultaneously at two wavelengths, 520 nm and 650 nm, each of which has a Doppler-broadened transition width $\approx$ 1.3 GHz. Calculate the number of resonant longitudinal modes that would be sustained by this laser if the resonant cavity were 260 mm long and the refractive index of the lasing medium were 1.53.

[6 marks]

Suggest one method that could be used to suppress the output of the shorter wavelength from this laser.

[4 marks]