King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP3380 Optics

Summer 2004

Time allowed: THREE Hours

Candidates should answer no more than SIX parts of SECTION A, and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_{e}	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_{u}	=	1.661×10^{-27}	kg
Proton rest mass	m_{p}	=	1.673×10^{-27}	kg
Neutron rest mass	m_{n}	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_{B}	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_{A}	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

SECTION A – Answer SIX parts of this section

1.1) State briefly how *Huygens' Principle* could be used to understand the propagation of an electromagnetic wave.

Identify an important modification to Huygens' Principle that was introduced by Fresnel.

[7 marks]

1.2) Write down an expression for the chromatic resolving power of a regular linear diffraction grating, defining all the symbols used.

Hence determine the minimum number of slits that would be needed for the grating to just resolve the two yellow sodium D lines, with wavelengths 588.995 nm and 589.592 nm, using the 5th-order principal maxima.

[7 marks]

1.3) A parallel beam of red light is incident on the boundary between a plane glass sheet of refractive index $n = 1.56$ and free space. Calculate the critical angle of incidence θ_c for total internal reflection of the red light.

Assuming the refractive index of the glass decreases with increasing wavelength, explain briefly whether or not blue light incident at the same angle θ_c would be totally reflected at the glass-air boundary.

[7 marks]

1.4) State the two principal ideas that underlie the Abbe theory of image formation. Use these ideas to explain why it is not normally possible for the image to be a faithful copy of the object in a single-lens optical system.

[7 marks]

1.5) An optical system has a pupil function defined by a circular aperture, and is used with monochromatic, spatially incoherent illumination. State the Rayleigh criterion for determining when the images of two point-like objects are just resolved by this optical system.

Use the Rayleigh criterion to estimate the spatial resolution of the optical system, given that it has a numerical aperture of 0.7, and the illumination is of wavelength 590 nm.

[7 marks]

1.6) An off-axis hologram is to be recorded with a mean angle between the reference and object beams of $90°$. The wavelength of the illumination is 450 nm, and the two beams are located symmetrically about the normal to the plane of the recording medium. Calculate the minimum resolution (in lines per mm) needed for the film that is used to record the hologram.

Explain briefly why is it essential either to use a system with a very high degree of mechanical stability, or to use a very short exposure time, when recording a hologram.

[7 marks]

1.7) Describe, with the aid of a diagram, how a *pseudoscopic* image can be formed when reconstructing a holographic image.

Explain briefly what is meant by the term *pseudoscopic*.

[7 marks]

1.8) Explain briefly why a population inversion is essential for laser operation. Describe, with the aid of a diagram, how a population inversion is achieved in a four-level laser.

[7 marks]

SECTION B – Answer TWO questions

2) Light from a monochromatic point source of wavelength λ is incident normally on an opaque screen in which there are eight very long rectangular slits (numbered 1 to 8), each of width a. The slits are parallel and their centres lie along the x axis, separated by a centre-to-centre distance d (where $d > a$).

a) Show that, in the far field, the diffraction pattern along an axis parallel to the x axis is given by an expression of the form

$$I(u) = I_o a^2 \left(\frac{\sin \pi u a}{\pi u a}\right)^2 \left(\frac{\sin 8\pi u d}{\sin \pi u d}\right)^2$$

[8 marks]

b) Comment briefly on the interpretation of the terms in this expression, and make clear the meaning of the symbols I_o and u.

[6 marks]

c) Sketch this function in the range $|u| \leq 10/d$ for the case $a = d/4$, commenting quantitatively on the main features of the diffraction pattern.

[8 marks]

d) Discuss how the diffraction pattern would change if slits 1, 3, 5 and 7 were covered by an opaque mask.

[5 marks]

e) Describe briefly the diffraction pattern you would expect to observe if slits 2, 4, 6 and 8 were covered by an opaque mask instead. Explain briefly why you would expect this pattern.

[3 marks]

3) The amplitude reflection coefficients for p and s polarised light incident on the boundary between two transparent dielectric materials can be written in the form

$$R_p = \frac{\tan(\theta_2 - \theta_1)}{\tan(\theta_2 + \theta_1)} \quad \text{and} \quad R_s = \frac{\sin(\theta_2 - \theta_1)}{\sin(\theta_2 + \theta_1)}$$

where subscripts p and s have their usual meanings, θ_1 represents the angle of incidence in a medium of refractive index n_1, and θ_2 represents the angle of refraction in a medium of refractive index n_2.

a) Linearly polarised light is incident on the boundary between the two media at angle θ_1. Assuming that $n_2 > n_1$:

i) Explain carefully why the plane of polarisation of the reflected light will generally be different from that of the incident light.

[6 marks]

ii) Show that, at one particular angle of incidence θ_1, the reflected light will be polarised perpendicular to the plane of incidence. Derive an expression for this angle θ_1 in terms of the refractive indices n_1 and n_2.

[7 marks]

iii) If the incident light is linearly polarised at 45° to the plane of incidence, show that the plane of polarisation of the reflected light is at an angle α with respect to the plane of incidence given by

$$\alpha = \tan^{-1}\left[\frac{\cos(\theta_2 - \theta_1)}{\cos(\theta_2 + \theta_1)}\right]$$

[4 marks]

b) By using Snell's law, show that the amplitude reflection coefficient for s polarised light can be written as

$$R_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

[3 marks]

Hence show that the intensity transmittance $\mathcal{T}$ for both p and s polarised light at normal incidence is given by

$$\mathcal{T} = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

[5 marks]

c) A scuba diver wears a clear perspex face-mask, of refractive index 1.49, while swimming underwater. Assuming the refractive indices of water and air are 1.33 and 1.0, respectively, estimate the fraction of incident light that is transmitted through the face-mask.

[5 marks]

4) Describe what is meant by *spatial filtering* in the context of image formation in an optical system.

[3 marks]

The transmission function for an object is given by $f(x) = \exp\left[i\,\phi_o \sin\left(2\pi x/d\right)\right]$, where ϕ_o and d are constants, and x is a coordinate perpendicular to the optical axis of the system.

a) What is the physical interpretation of the terms ϕ_o and d?

[2 marks]

b) Explain why there would be no image contrast if the object was imaged using a single thin converging lens, with incoherent illumination.

[5 marks]

The light source is now replaced by one that illuminates the object with a coherent monochromatic plane wave travelling parallel to the optical axis of the lens.

c) Briefly describe one further change to the optical system that would allow meaningful image contrast to be produced.

[6 marks]

d) Given that $\phi_o \ll 1$, show that the complex amplitude distribution in the back focal plane of the imaging lens is given by an expression of the form

$$E(u) \sim E_0 \left[\delta(u) + \frac{\phi_o}{2}\left(\delta\left(u - 1/d\right) - \delta\left(u + 1/d\right)\right)\right]$$

where $\delta()$ denotes the Dirac delta function. State clearly any assumptions that you have made when deriving the expression for $E(u)$.

[6 marks]

e) If the illumination is of wavelength $\lambda = 600\,\text{nm}$, the lens has a focal length of $250\,\text{mm}$, and $d = 36\,\mu\text{m}$, calculate the separation of the diffraction spots in the back focal plane of the lens.

[4 marks]

f) A circular aperture of diameter $20\,\text{mm}$ is added to the back focal plane of the lens, and centred on the optical axis, and the space between the lens and the object is filled with a transparent oil of refractive index 1.55. Calculate the effective numerical aperture of this optical system, and the minimum spatial period that could be resolved in the object.

[4 marks]

5) With the aid of a diagram, explain how a multi-layer mirror can achieve a very high reflectivity for a narrow range of wavelengths.

[10 marks]

Suggest two reasons why multi-layer mirrors might be a good choice for use as the end reflectors in the resonant cavity of a laser system.

[4 marks]

Explain how a very short resonant cavity in a laser system can result in light output from the laser that has a narrower range of wavelengths than might be expected from the natural linewidth of the atomic transition involved in the lasing action.

[6 marks]

A gas laser system with an 80 cm long resonant cavity has laser emission at a wavelength of 543 nm, with a Doppler-broadened transition width $\Delta\nu \approx 1.5\,\text{GHz}$. Assuming the gas refractive index $n = 1$, determine the number of resonant longitudinal modes that can be sustained by this laser and calculate the maximum cavity length that would ensure single-mode operation of the laser.

[6 marks]

If the linewidth of a single resonant mode is 2.5 MHz, estimate the coherence time and longitudinal coherence length for the laser output when it is operated in single mode form.

[4 marks]