King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3380 Optics

Summer 2002

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck constant	h	$= 6.626 \times 10^{-34}\,\mathrm{J\,s}$

SECTION A – Answer SIX parts of this section

1.1) A beam of light is incident on the plane boundary between two uniform transparent media, of refractive index n_1 and n_2 respectively. State the relation between the angle of incidence θ_1 and the angle of refraction θ_2 that applies only when the angle of incidence is equal to the *Brewster* angle.

If the Brewster angle is 53°, and $n_1 = 1.33$, calculate the value of the refractive index n_2.

[7 marks]

1.2) With the aid of a diagram, explain briefly how a Michelson interferometer could be used to measure the temporal coherence length of a beam of light.

[7 marks]

1.3) Explain how the convolution theorem for Fourier transforms can be of use when calculating the Fraunhofer diffraction pattern from a set of identical apertures.

[7 marks]

1.4) Write down an expression for the far-field diffracted intensity from an object with transmission function $f(x, y)$ when it is illuminated by a monochromatic plane wave propagating along a normal to the object plane.

By considering two diffracting objects with transmission functions $f(x, y)$ and $1 - f(x, y)$ respectively, outline how Babinet's principle, describing the relation between the Fraunhofer diffraction patterns from a complementary pair of masks and apertures, may be deduced.

[7 marks]

1.5) State what is meant by *phase contrast imaging.*

Identify two important differences between the Schlieren and the Zernike methods of phase contrast imaging.

[7 marks]

1.6) An optical system has a pupil function defined by a circular aperture, and is used with monochromatic, spatially incoherent illumination. State the Rayleigh criterion for determining when the images of two point-like objects are just resolved by this optical system.

[7 marks]

1.7) Comment briefly on any differences you might notice in the reconstructed holographic image from a reflection hologram when you view it in daylight, first in direct sunlight, and then on an overcast cloudy day.

[7 marks]

1.8) With the aid of a diagram, explain how a population inversion can be achieved in a 3-level laser.

Identify two of the main disadvantages of using a 3-level system.

[7 marks]

SECTION B – Answer TWO questions

2(a) Use a diagram to outline the principles involved in recording an off-axis (or sideband) Fresnel hologram. Indicate how the holographic image could be reconstructed, and explain what is meant by the term *pseudoscopic image*.

[12 marks]

Identify two advantages of the off-axis geometry for recording a Fresnel hologram when compared to the in-line (or Gabor) geometry.

[4 marks]

(b) Suggest two reasons why a resonant cavity is frequently used as part of a laser.

[6 marks]

For a particular laser, it is found that stimulated emission occurs simultaneously at two wavelengths, 488 nm and 647 nm, each of which has a Doppler-broadened transition width $\approx 1.8\,\text{GHz}$. Calculate the number of resonant longitudinal modes that would be sustained by this laser if the resonant cavity was 350 mm long and the refractive index of the lasing medium was 1.0.

[6 marks]

Suggest one method that could be used to suppress the output of the shorter wavelength from this laser.

[2 marks]

3) The far-field diffracted intensity from a regular one-dimensional grating of N identical apertures of width a, separated by a centre-to-centre distance d, is of the form

$$I(u) = I_0 \left[\frac{\sin^2(\pi a u)}{(\pi a u)^2} \right] \left[\frac{\sin^2(N \pi d u)}{\sin^2(\pi d u)} \right]$$

Explain the physical significance of each of the three terms on the right hand side of this equation. Write down an expression for the chromatic resolving power of this grating, defining all symbols used.

[10 marks]

Figure 3.1 shows a plot of the diffracted intensity from a regular one-dimensional array of N identical slits of width a that have a centre-to-centre separation of $d = 360\,\mu$m. Using the information on the figure, determine: (a) the number N of apertures in the array, (b) the width a of each of the apertures in the array, and (c) whether the 8th-order principal maximum could be used to resolve two wavelengths of value 600 nm and 616 nm.

[12 marks]

Explain briefly whether the following statements are true or false:

(i) The angular separation of the principal maxima in the Fraunhofer diffraction pattern from the grating will decrease as the wavelength of the illumination is increased.

(ii) The chromatic resolving power of the diffraction grating will increase as the wavelength of the illumination is increased.

[8 marks]

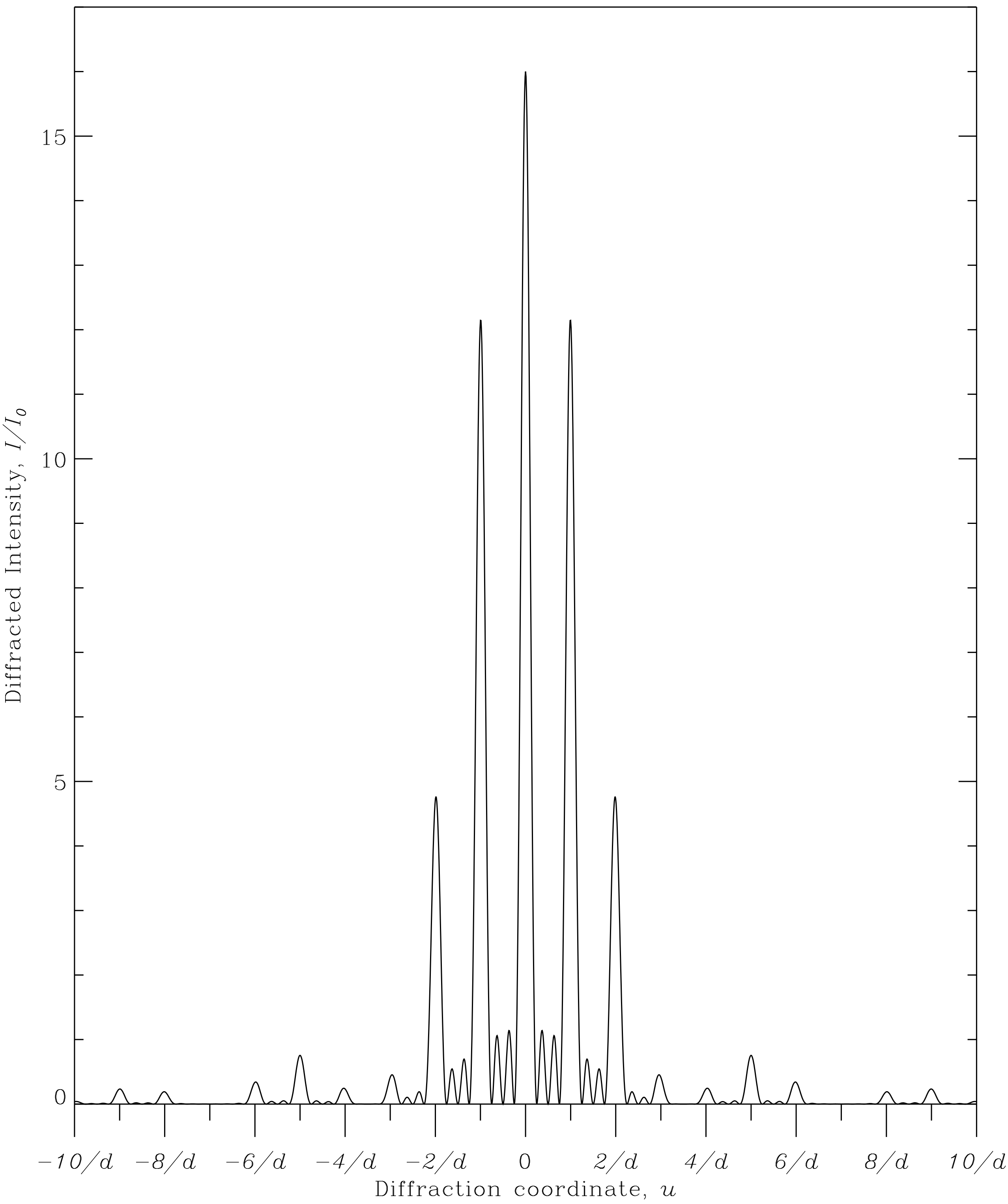

Figure 3.1 Diffracted intensity from a regular one-dimensional array of N identical slits.

4) The optical forms of the amplitude transmission coefficients for p and s polarised light are

$$T_p = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1} \qquad \text{and} \qquad T_s = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_2 + n_1 \cos\theta_1}$$

respectively, where subscripts p and s have their usual meanings, θ_1 represents the angle of incidence in a medium of refractive index n_1, and θ_2 represents the angle of refraction in a medium of refractive index n_2.

The incident light is plane polarised with the direction of polarisation at an angle of 45° relative to the plane of incidence. Calculate the direction of polarisation for the transmitted light in the case where $\theta_1 = 60°$, $n_1 = 1.0$, $n_2 = 1.6$.

[8 marks]

Explain briefly why the direction of polarisation for the transmitted light will be less than 45° for all angles of incidence except $\theta_1 = 0$.

[3 marks]

Show explicitly that when $\theta_1 = 0$, the intensity transmittance $\mathcal{T}$ can be written as

$$\mathcal{T} = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

and hence deduce an expression for the intensity reflectance at normal incidence.

[8 marks]

A sample holder in a spectrometer has plane glass windows of refractive index 1.76, and the light is incident normally on these windows, travelling from free space with refractive index 1.0. Calculate the intensity transmittance through the sample holder when it is empty, and when it is filled with the transparent liquid benzene, of refractive index 1.50.

[8 marks]

Explain why the two values are different.

[3 marks]

5) State the two principal ideas underlying Abbe's theory of image formation. Use these ideas to explain qualitatively how a small axial aperture in the back focal plane of a lens can limit the resolution of an image formed by the lens.

[7 marks]

A coherent monochromatic plane wave of wavelength 450 nm is travelling parallel to the optical axis of a thin converging lens of focal length 15 cm and diameter 4 cm. An object is placed on the optical axis a distance $s_1 > 15\,\text{cm}$ from the lens, and the image is observed as the object distance is changed. Explain why the image resolution becomes worse as the object distance s_1 increases.

When the object is located 40 cm before the lens, calculate the minimum diameter of a circular aperture that could be centred on the optical axis, in the back focal plane of the lens, without affecting the image resolution.

[6 marks]

A new object is positioned at a distance $s_1 = 15\,\text{cm}$ in front of (*i.e.,* upstream from) the lens described above, and a second identical lens is mounted coaxially with the first one, at a position 30 cm downstream from the first lens. The object has a transmission function of the form

$$f(x) = [0.6 + 0.12\cos(2\pi x/d)]$$

where $d = 0.09\,\text{mm}$ and x is a direction perpendicular to the optical axis. Show that, when the illumination is the same as described above, there will be three small bright spots in the back focal plane of the first lens. Calculate the distance between these spots.

[8 marks]

An absorbing mask is now placed in the back focal plane of the first lens, so that it attenuates the *intensity* of the central spot by a factor of 9, and leaves the other two spots unaffected. Derive an expression for the intensity distribution in the image plane of the second lens, and show that the intensity contrast in the image plane is a factor of 2.29 higher than the intensity contrast in the object plane.

[9 marks]