King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3380 Optics

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light in free space $c = 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck constant $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$

SECTION A – Answer SIX parts of this section

1.1) State Snell's law for refraction at the boundary between two transparent dielectric media, defining all variables used.

Calculate the critical angle for total internal reflection when light is incident on the boundary between a plane glass sheet of refractive index $n_1 = 1.72$ and liquid paraffin of refractive index $n_2 = 1.43$. Draw a clearly-labelled diagram showing the ray paths when a parallel beam of light is incident on the boundary at the critical angle.

[7 marks]

1.2) State *Fermat's principle* as it applies in ray optics. Show how Fermat's principle can be used to derive the law of reflection for light reflected at the plane boundary between two uniform media.

[7 marks]

1.3) A filter is used to obtain approximately monochromatic yellow light from a white light source. If the filter transmits wavelengths from 580 nm to 600 nm, estimate the coherence time and coherence length of the filtered light.

[7 marks]

1.4) A single narrow slit is illuminated with coherent monochromatic light. With the aid of some sketches, describe qualitatively how the Fresnel diffraction pattern from the slit changes as the slit width is increased from a setting that is initially very narrow. Comment very briefly on the changes that would occur if the Fraunhofer diffraction pattern of the slit had been observed instead.

[7 marks]

1.5) Describe qualitatively the appearance of the Fraunhofer diffraction pattern from a single circular aperture of diameter a, when it is illuminated by a coherent monochromatic plane wave of wavelength $\lambda \ll a$. Describe how the diffraction pattern would change if N identical circular apertures were located at random positions in the same plane as the original aperture, assuming that N is a large number.

[7 marks]

1.6) State what is meant by *phase contrast imaging.*

Draw a well-labelled diagram that illustrates the Schlieren method of phase contrast microscopy.

[7 marks]

1.7) Identify three advantages of the off-axis geometry for recording a Fresnel hologram when compared to the in-line (or Gabor) geometry.

[7 marks]

1.8) Explain briefly what is meant by the term *population inversion* in the context of laser physics. Show that the existence of a population inversion in a two-level atomic system is sufficient to ensure net emission of light from the system.

[7 marks]

SECTION B – Answer TWO questions

2) A monochromatic plane wave of light of wavelength λ is incident normally on a plane diffraction grating with N rectangular slits of width a and length b, separated by a centre-to-centre distance d (where d is a distance parallel to width a, and $d > a$). The diffracted intensity in the far-field is found to have the form

$$I_o \left(\frac{\sin^2 \alpha}{\alpha^2}\right) \left(\frac{\sin^2 \beta}{\beta^2}\right) \left(\frac{\sin^2 N\gamma}{\sin^2 \gamma}\right)$$

Identify the terms α, β and γ, and discuss briefly *(without detailed derivation)* the origin of each of the bracketed terms in this expression.

[9 marks]

Show that the chromatic resolving power of the grating around the mth principal maximum in the diffraction pattern is given by $\lambda/\delta\lambda = mN$.

[9 marks]

A linear grating with 68 rectangular slits is illuminated by green light from a discharge lamp. This light consists of two closely spaced wavelengths with a mean wavelength of 517.81 nm. A series of bright diffraction spots are observed in the diffraction pattern 2.5 m from the grating. The diffraction spots near the centre of the pattern are evenly spaced with a mean separation of 3 mm in the diffraction plane, except that on moving away from the centre of the pattern every 9th spot is absent. When the 7th order and higher order spots are studied carefully it is seen that there are two spots very close together.
Determine the values of a and d for this grating, and calculate the difference in the wavelengths of the two green lines.

[12 marks]

3) For light polarised *parallel* to the plane of incidence that is the incident on the boundary between two media of refractive indices n_1 and n_2 respectively, the amplitude reflection coefficient for light at an incident angle θ_1 in medium 1 is given by

$$R_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} \tag{3.1}$$

where θ_2 is the angle of refraction in medium 2. The *Brewster* angle of incidence is defined by the condition

$$\theta_1 + \theta_2 = \frac{\pi}{2}.$$

Hence find the value of R_p at the Brewster angle of incidence.

[6 marks]

Explain how Brewster windows are used in some types of laser. Calculate the Brewster angle for the windows of a gas laser when the gas has refractive index 1, and the windows are of fused silica, with refractive index 1.46.

[8 marks]

Use equation (3.1) to derive an expression for the intensity reflectance for light incident normally on to the boundary between the two transparent dielectric media, and hence deduce that the intensity transmittance at normal incidence is given by

$$T_I = \frac{4n_2 n_1}{(n_2 + n_1)^2}$$

[4 marks]

Light emitted by a scintillating crystal of sodium iodide (refractive index 1.77) is incident normally on the plane entrance window of a photomultiplier tube (PMT). The entrance window is made of fused silica; it is in good optical contact with the scintillating crystal, and there is free space between the window and the photocathode of the PMT. Calculate the fraction of the light that is transmitted through the window.

Determine also the consequence of having an air gap between the scintillator and the PMT (assuming the refractive index of the air is 1).

Suggest one way of increasing the fraction of light that will be transmitted through the detector window.

[12 marks]

4) With the aid of a diagram, explain briefly why it is easier to sustain continuous light output from a four-level laser than from a three-level laser.

[6 marks]

Describe three benefits that the use of a resonant cavity can bring to the successful operation of a laser. List three factors that can impair the performance of the laser, and suggest ways of reducing their effect.

[9 marks]

Show that for a resonant cavity of length L the frequency difference between consecutive longitudinal resonant modes is $\Delta\nu = c/2nL$, where c is the free-space velocity of light, and n is the refractive index of the medium within the cavity.

[3 marks]

For a particular laser, it is found that stimulated emission occurs simultaneously at two wavelengths, 440 nm and 580 nm, each of which has a Doppler-broadened transition width $\approx$ 1.8 GHz. Calculate the number of resonant longitudinal modes that would be sustained by this laser if the resonant cavity is 150 mm long and the refractive index of the lasing medium is 1.34.

[6 marks]

What change would you make to the laser cavity to ensure that only one resonant longitudinal mode was sustained by the laser at 440 nm, and one mode at 580 nm? Suggest one method that could then be used to suppress the output of the shorter wavelength from this laser.

[6 marks]

5a) State briefly the two principal ideas that form the basis of the Abbe theory of image formation.

[4 marks]

Describe what is meant by *spatial filtering* in the context of image formation in an optical system.

[4 marks]

Consider the case of an object that is weakly absorbing and has an amplitude transmission function $f(x, y) = 1 - a(x, y)$, where $a(x, y) \geq 0$ and $a(x, y) \ll 1$. The optical system has a pupil function given by

$$P_1(u, v) = \begin{cases} 1 & \text{if } u = v = 0, \\ -1 & \text{elsewhere.} \end{cases}$$

Derive an expression for the image intensity distribution. Comment briefly on any difference you would observe between the image formed in this way and one formed when the optical system has the pupil function $P_2(u, v) = 1$ for all values of (u, v).

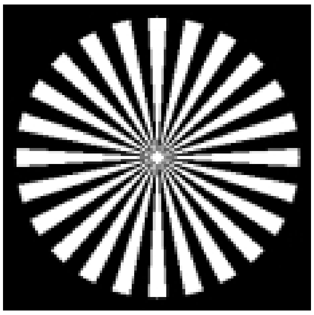
[6 marks]

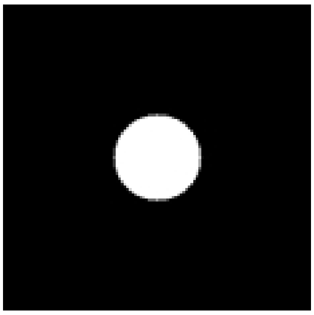
If the physical mask corresponding to pupil function $P_1(u, v)$ is to be made from a single sheet of transparent plastic, of refractive index $n = 1.4$, calculate a suitable thickness for the plastic sheet when the wavelength of the illumination is 640 nm. Comment briefly on any practical difficulties in realising the mask.

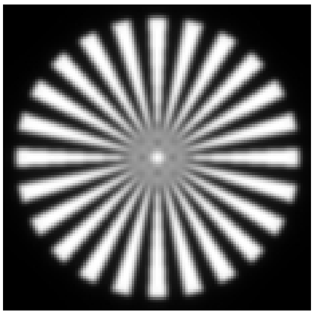
[6 marks]

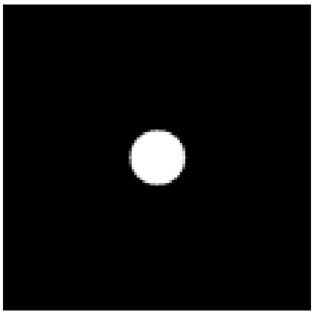
b) The 2-dimensional object whose intensity transmission function is shown in Figure 5.i is illuminated with a *spatially incoherent* monochromatic plane wave, and a number of different masks are placed in the back focal plane of the first lens to limit the numerical aperture of the optical system. The masks are shown in Figure 5.ii and the five image intensity distributions that result are shown in Figure 5.iii. Identify which of the images could be associated with each of the masks shown, briefly stating a reason for each of your choices.

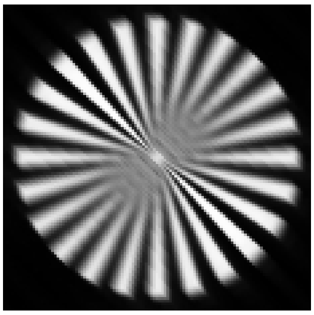
[10 marks]

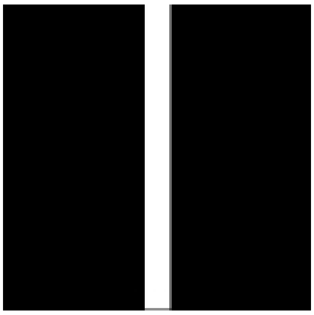

5.i Object

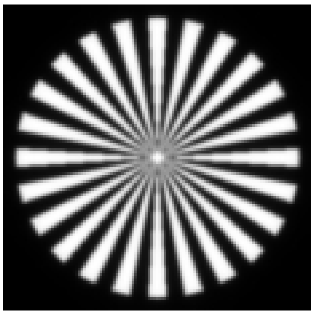

5.ii Mask 1

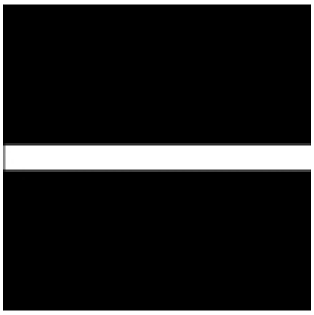

5.iii Image A

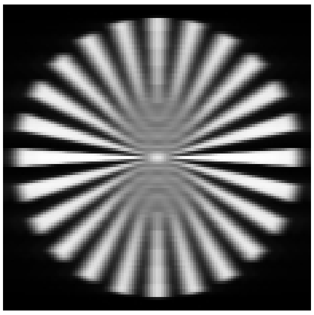

5.ii Mask 2

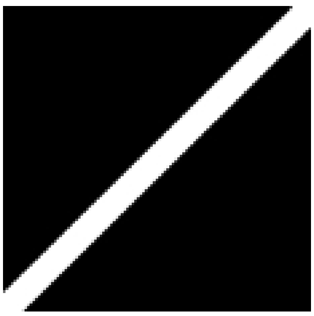

5.iii Image B

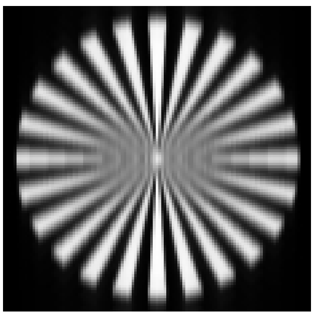

5.ii Mask 3


5.iii Image C


5.ii Mask 4


5.iii Image D


5.ii Mask 5


5.iii Image E