King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3380 Optics

Summer 2000

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck constant	h	$= 6.626 \times 10^{-34}\,\mathrm{J\,s}$

SECTION A – Answer SIX parts of this section

1.1) A monochromatic beam of light is incident normally on the surface of a plane sheet of glass of refractive index n. Explain how the addition of a thin transparent film to the surface of the glass sheet can be used to reduce the intensity of the light reflected from the surface.

[7 marks]

1.2) With the aid of a diagram, distinguish between the *spatial* and *temporal* coherence of a beam of light.
State how the coherence *length* can be related to the coherence *time.*

[7 marks]

1.3) State briefly how *Huygens' Principle* could be used to understand the propagation of an electromagnetic wave.

Identify an important modification to Huygens' Principle that was introduced by Fresnel.

[7 marks]

1.4) A clear plastic sheet coated with a thin layer of transparent adhesive is used to the collect the soot particles present in diesel exhaust fumes. Explain briefly how a Fraunhofer diffraction experiment could be used to measure the approximate size of the soot particles, stating clearly any assumptions that you make.

[7 marks]

1.5) Write down an expression for the *numerical aperture* of an imaging system, defining all symbols that are used.

Explain why an imaging system with a small numerical aperture is expected to have poorer spatial resolution than one that has a large numerical aperture.

[7 marks]

1.6) Distinguish between the recording conditions needed to produce an *in-line (or Gabor) Fresnel hologram* and a *Fourier-transform hologram* of a thin transmitting object.

[7 marks]

1.7) A regular linear diffraction grating is to be produced 'holographically' in a sheet of photographic emulsion, using the interference of two beams of visible light. Outline the main experimental requirements for this process to be successful.

If the grating is required to have 2500 lines per millimetre, and the two light beams are directed at angles of $+30°$ and $-30°$ to the film normal, calculate the wavelength that should be used for the illumination.

[7 marks]

1.8) Explain briefly whether the following statements are true or false.

a) A 4-level laser can operate successfully without the need for a population inversion, since only a small fraction of the atoms need to be excited out of the ground state.

b) When stimulated emission is observed from a medium in which there is a population inversion, the light will all have the same polarisation.

[7 marks]

SECTION B – Answer TWO questions

2) State *Fermat's principle* as it applies in ray optics. Show how Fermat's principle can be used to derive Snell's law of refraction for light incident at the plane boundary between two uniform media of refractive index n_1 and n_2 respectively.

[10 marks]

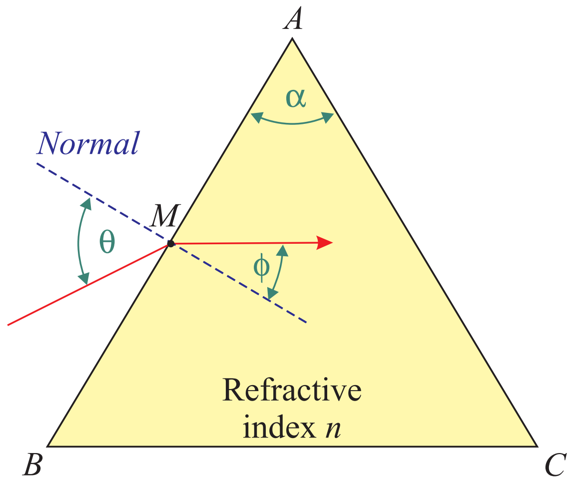

Figure a: Top view of prism ABC

A triangular prism ABC is made from glass of refractive index n. The rectangular faces of the prism lie in a vertical plane, and a horizontal ray of light strikes surface AB at its midpoint M, making an angle θ with the normal to the surface. Assume that the base of the prism is an equilateral triangle, and that the prism is surrounded by free space.

a) Show that when the ray refracted inside the prism travels parallel to the face BC, as shown in Figure a, the refractive index is given by the relation $n = 2\sin\theta$. If the total deviation of this ray after it emerges from side AC is $54.28°$, show that the refractive index of the prism is $n = 1.68$.

[10 marks]

b) If the direction of the incident ray is kept fixed while the prism is rotated about a vertical axis through the point M, explain briefly whether you would expect the total angle of deviation to increase or decrease for rotations of a few degrees (i) clockwise and (ii) anti-clockwise, from the inital position shown in Figure a.

[4 marks]

If the prism in Figure a is immersed in a fluid of refractive index $n_2 = 1.1n$, find the minimum angle of incidence θ that would prevent any light incident on surface AB being transmitted through the prism.

[6 marks]

3) A diffraction grating consists of a regular linear array of N identical rectangular apertures of width a and height b, where $b \gg a$. The apertures are located on the x-axis, separated by a centre-to-centre distance $d = 5a$.
In the Fraunhofer approximation, the diffracted intensity can be written as the product of a constant term, a *diffraction envelope* and an *interference term*. Show that the diffraction envelope has the form

$$(ab)^2 \operatorname{sinc}^2(\pi a u) \operatorname{sinc}^2(\pi b v)$$

where u, v are diffraction coordinates parallel to the x and y axes respectively, and show that the interference term has the form

$$\left(\frac{\sin(N\pi d u)}{\sin(\pi d u)}\right)^2$$

[12 marks]

Derive an expression for the condition that must be satisfied when 'missing orders' occur, and identify which orders will be missing for the grating described above.

[6 marks]

The grating is replaced by one that has circular apertures of diameter a instead of rectangular slits, although the centre-to-centre separation d is the same as before. Describe briefly *(without detailed calculation)* the *qualitative* differences you would see in the form of the diffracted intensity, and explain why fewer diffracted orders are now 'missing'.

[8 marks]

If there are 600 apertures in the grating, determine which diffraction order must be used if the yellow helium doublet is to be resolved, given that the doublet separation is 0.035 nm, with a mean wavelength of 587.580 nm.

[4 marks]

4) State briefly the two principal ideas that form the basis of the Abbe theory of image formation, and explain why the concept of a *pupil function* is useful when analysing the behaviour of a lens-based imaging system.

[6 marks]

A phase object has transmission function $f(x, y) = \exp[i\phi(x, y)]$, where $\phi(x, y)$ is a real function and $\phi(x, y) \ll 1$. Outline the principle behind the Zernike method of phase contrast imaging, and, with the aid of a sketch, describe a suitable optical system that would allow you to form an image of this object using the Zernike method. Suggest at least 2 ways in which the optical system you describe would differ from the theoretically ideal form of the Zernike technique.

[9 marks]

Write down a mathematical expression for the pupil function used in the theoretically ideal case of the Zernike technique, and use this to show that the intensity in the image plane is proportional to either $1 + 2\phi(x, y)$ or $1 - 2\phi(x, y)$ (depending on your choice of pupil function). Describe the change in the pupil function that is needed to reverse the sign in front of $\phi(x, y)$.

[8 marks]

Assuming that the function $\phi(x, y)$ is given by the expression

$$\phi(x, y) = A\left[0.5 + \sin^2(3x + 4y)\right] \qquad \text{(where } A \text{ is a constant)},$$

show that the intensity contrast in the image plane, when using the ideal form of the Zernike method, is

$$\frac{A}{1 + 2A}$$

[4 marks]

If the Zernike filter is removed from the optical system, and the illumination is incoherent, explain why the image contrast is then zero.

[3 marks]

5) (a) State the main principle involved in the holographic recording of an image. Suggest two reasons why a laser is so commonly used as the source of illumination when recording a hologram with visible light.

[5 marks]

With the aid of a diagram, describe how a 'white light' or reflection hologram of a reflecting object could be recorded. Explain how the holographic image can be viewed, and comment briefly on the advantages and disadvantages of this form of holographic imaging.

[10 marks]

(b) Describe briefly how the operation of a laser benefits from enclosing the lasing medium in a resonant cavity.

[6 marks]

A gas laser system with a 60 cm long resonant cavity has laser emission at wavelengths of 488 nm and 647 nm, each with a Doppler-broadened transition width $\Delta\nu \approx 1.0\,\mathrm{GHz}$. Assuming the gas refractive index $n = 1$, determine the number of resonant longitudinal modes that can be sustained by this laser and calculate the maximum cavity length that would ensure single-mode operation of the laser at each wavelength. Suggest one way in which the output of the shorter wavelength could be suppressed.

[9 marks]