King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/3240 Theoretical nuclear and particle physics

Summer 1998

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Rest mass of electron $m_e = 0.511\,\text{MeV}\,c^{-2}$
Rest mass of muon $m_\mu = 105.660\,\text{MeV}\,c^{-2}$
Rest mass of charged pion $m_\pi = 139.568\,\text{MeV}\,c^{-2}$
Rest mass of proton $m_p = 1.673\times10^{-27}$ kg
Rest mass of top quark $m_t \approx 175\,\text{GeV}\,c^{-2}$
Reduced Planck constant $\hbar = 1.055\times10^{-34}$ J s
Speed of light $c = 2.998\times10^{8}\,\text{m s}^{-1}$
Electron volt, 1 eV $= 1.602\times10^{-19}$ J

SECTION A. Answer any **SIX** parts of this section.

1.1 Which of the following reactions are allowed to proceed by way of the weak interaction?

(i) $\nu_e + p \rightarrow e^+ + n$
(ii) $\nu_\mu + n \rightarrow \mu^- + \pi^0 + p$
(iii) $\Sigma^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$
(iv) $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$.

Give a reason for each reaction which is forbidden.

[7 marks]

1.2 State which force is most likely to mediate the following processes:

(i) $e^- + p \rightarrow \nu_e + n$
(ii) $e^- + e^- \rightarrow e^- + e^-$
(iii) $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$
(iv) $e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu$.

Draw an appropriate Feynman diagram in each case.

[7 marks]

1.3 A charged pion decays at rest into a muon and a neutrino (e.g., $\pi^+ \rightarrow \mu^+ + \nu_\mu$). The energy of the muon is measured to be *at least* 109.450 MeV. Show that the rest mass of the neutrino is $m_\nu \leq 9.583\,\text{MeV}\,c^{-2}$.

[7 marks]

1.4 Write down one example of the associated production of strange particles which occurs when a proton beam is made to collide with a solid target. Which force is involved in this process? By which force do most strange particles decay? Write down one example of this, and one example of a strange particle decay mediated by a different force.

[7 marks]

1.5 How did the *Cabibbo angle* help to explain the observed β-decay rates of strange particles? When comparing the rates of lambda and neutron β-decays it is found that

$$\frac{\text{rate}\,(\Lambda \rightarrow p + e^- + \bar{\nu}_e)}{\text{rate}\,(n \rightarrow p + e^- + \bar{\nu}_e)} = 0.053.$$

Determine a value for the Cabibbo angle.

[7 marks]

1.6 What are the principal components of a modern particle accelerator? In such machines, why are the particles accelerated in bunches?

[7 marks]

1.7 What are the symmetries related to the conservation of energy, momentum and angular momentum? Show that the parity operation is equivalent to a mirror reflection followed by a rotation. Why is it possible to ignore the rotation when considering the effect of the parity operation on a system?

[7 marks]

1.8 What is meant by the *three generations of matter*? State one piece of evidence which indicates that there is not a fourth generation.

[7 marks]

SECTION B. Answer any **TWO** questions from this section.

2 a) Define what is meant by (i) the four-momentum of a particle, (ii) a conserved quantity and (iii) an invariant quantity. Give one example of a conserved quantity and one of an invariant quantity.

[8 marks]

b) Define the system of natural units. Hence obtain values, in SI units, for the natural units of length, energy and time.

[6 marks]

c) Show that when a particle of mass m collides with a stationary particle of mass M, the minimum energy required to produce a state of mass M^* is, in natural units,

$$E_{\text{threshold}} = \frac{M^{*2} - M^2 - m^2}{2M}.$$

[6 marks]

d) Determine the threshold energy for producing a meson containing the top and anti-top quarks when beams of equal energy electrons and positrons are made to collide head on.

[5 marks]

e) Estimate the beam energy required in a fixed target electron accelerator to achieve the same centre-of-mass energy as in part (d).

[5 marks]

3 a) Write down the general quark structures of baryons and mesons.

[2 marks]

b) State why the quark structures of some baryons caused an initial problem with the quark model, and describe how this problem was overcome by the introduction of the quantum number *colour*.

[4 marks]

c) Briefly describe the experiment in which the bottom quark was discovered. Why did the subsequent discovery of the top quark entail a much more difficult experiment?

[6 marks]

d) Five observed hadrons have the following quantum numbers (Q, B, S, C, $\underline{B}$, T):

(i) (–1, +1, –3, 0, 0, 0)
(ii) (+1, +1, –1, +1, 0, 0)
(iii) (0, 0, 0, –1, 0, 0)
(iv) (0, +1, 0, 0, –1, 0)
(v) (0, –1, +2, 0, 0, 0).

Identify the quark constituents of each of these hadrons, explaining your reasoning.

[10 marks]

e) Hadrons with the following quantum numbers (Q, B, S, C, $\underline{B}$, T):

(i) (+2, +1, 0, 0, 0, +1)
(ii) (–1, +1, 0, 0, 0, +1)
(iii) (0, 0, 0, –1, 0,+1)
(iv) (+1, 0, +1, 0, 0, –1)

have not yet been observed. Determine which of these states are compatible with the quark model and which are not.

[8 marks]

For parts (c) and (d) of question 3 you may find the following table helpful.

Name & symbol	Electric charge Q	Baryon number B	Strangeness S	Charm C	Bottomness $\underline{B}$	Topness T
Down, d	$-^1/_3$	$+^1/_3$	0	0	0	0
Up, u	$+^2/_3$	$+^1/_3$	0	0	0	0
Strange, s	$-^1/_3$	$+^1/_3$	–1	0	0	0
Charmed, c	$+^2/_3$	$+^1/_3$	0	+1	0	0
Bottom, b	$-^1/_3$	$+^1/_3$	0	0	–1	0
Top, t	$+^2/_3$	$+^1/_3$	0	0	0	+1

4 a) State and define the properties of a complete set of symmetry operations.

[8 marks]

b) Illustrate your answer to part (a) by considering the set of spatial symmetry operations which can be applied to an equilateral triangle.

[8 marks]

c) The table below gives the Clebsch-Gordon coefficients for combining two states with angular momenta $j_1 = 1$ and $j_2 = \frac{1}{2}$.

m_1	m_2	J: $^3/_2$, M: $+^3/_2$	J: $^3/_2$, M: $+^1/_2$	J: $^1/_2$, M: $+^1/_2$	J: $^3/_2$, M: $-^1/_2$	J: $^1/_2$, M: $-^1/_2$	J: $^3/_2$, M: $-^3/_2$
+1	$+^1/_2$	1					
+1	$-^1/_2$		$\sqrt{1/3}$	$\sqrt{2/3}$			
0	$+^1/_2$		$\sqrt{2/3}$	$-\sqrt{1/3}$			
0	$-^1/_2$				$\sqrt{2/3}$	$\sqrt{1/3}$	
−1	$+^1/_2$				$\sqrt{1/3}$	$-\sqrt{2/3}$	
−1	$-^1/_2$						1

Use this table to determine the relative amplitudes of the states $\left|J, M\right\rangle$ which result from the following combination of the states:

(i) $\left|j_1, m_1\right\rangle = \left|1, +1\right\rangle$ and $\left|j_2, m_2\right\rangle = \left|\frac{1}{2}, +\frac{1}{2}\right\rangle$

(ii) $\left|j_1, m_1\right\rangle = \left|1, -1\right\rangle$ and $\left|j_2, m_2\right\rangle = \left|\frac{1}{2}, +\frac{1}{2}\right\rangle$

(iii) $\left|j_1, m_1\right\rangle = \left|1, 0\right\rangle$ and $\left|j_2, m_2\right\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$.

[8 marks]

d) Use the table of Clebsch-Gordon coefficients to determine the relative amplitudes of the $\left|j_1, m_1\right\rangle$ and $\left|j_2, m_2\right\rangle$ states that contribute to the state $\left|J, M\right\rangle = \left|\frac{3}{2}, -\frac{1}{2}\right\rangle$.

[3 marks]

e) The state $\left|J, M\right\rangle = \left|2, 0\right\rangle$ is produced by combining two states each with $j = 1$ (i.e., $j_1 = j_2 = 1$). How many $\left|j_1, m_1\right\rangle\left|j_2, m_2\right\rangle$ combinations contribute to this $\left|J, M\right\rangle$ state?

[3 marks]

5 Write notes on *two* of the following:

a) Could the particles currently considered to be fundamental — quarks, leptons and gauge bosons — have substructure?

[15 marks]

b) The Higgs mechanism.

[15 marks]

c) Grand unified theories and supersymmetry.

[15 marks]