King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/3240 Theoretical nuclear and particle physics

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light $c = 2.998\times10^{8}\ \mathrm{m\,s^{-1}}$
Reduced Planck constant $\hbar = 1.055\times10^{-34}\ \mathrm{J\,s}$
Rest mass of electron $m_{\mathrm{e}} = 0.511\ \mathrm{MeV\,c^{-2}}$
Rest mass of proton $m_{\mathrm{p}} = 1.673\times10^{-27}\ \mathrm{kg} = 938.3\ \mathrm{MeV\,c^{-2}}$
Rest mass of top quark $m_{\mathrm{t}} \approx 175\ \mathrm{GeV\,c^{-2}}$
Rest mass of π^- $m_{\pi} = 139.6\ \mathrm{MeV\,c^{-2}}$
Rest mass of Σ^+ $m_{\Sigma} = 1189.4\ \mathrm{MeV\,c^{-2}}$

SECTION A – Answer SIX parts of this section

1.1) Which properties define whether a particle is a *hadron* or a *lepton*? State whether the following particles are hadrons, leptons or neither: electron, pion, photon, neutron.

[7 marks]

1.2) By considering the relevant conservation laws determine which of the following decays are allowed by first-order processes.

(i) $\Xi^0 \to \Lambda + \pi^0$

(ii) $\Xi^0 \to p + \pi^-$

(iii) $K^+ \to \mu^+ + \nu_\mu$

(iv) $K^0 \to \mu^+ + \mu^-$

For those which are allowed, state which interaction is involved. For those which are not allowed, state a conservation law which is violated.

[7 marks]

1.3) Draw and fully label all the possible first-order Feynman diagrams for each of the following processes.

(i) $\nu_e + e^- \to \nu_e + e^-$

(ii) $\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$

(iii) $\nu_\mu + e^- \to \nu_\mu + e^-$

(iv) $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$

[7 marks]

1.4) A particle X decays at rest into a Σ^+ and a π^-. Each of the decay products has a momentum of $152\ \mathrm{MeV\,c^{-1}}$. Calculate the mass of X. Given that the decay is mediated by the strong interaction, determine as many of the quantum numbers of X as possible and deduce a possible quark content.

[7 marks]

1.5) The discovery of weak neutral currents allowed (a) the removal of divergences in the calculation of weak interaction cross sections and (b) the unification of the weak and electromagnetic interactions. Briefly explain these two statements.

[7 marks]

1.6) Show, by use of an appropriate diagram, how *phase stability* leads to bunching of charged particles in a cyclic accelerator such as a synchrotron.

[7 marks]

1.7) What are the symmetries related to the conservation of energy, momentum and angular momentum? Define the parity operation and show, with the aid of a diagram, that it is equivalent to a mirror reflection followed by a rotation.

[7 marks]

1.8) How did the *Cabibbo angle* help to explain the observed β-decay rates of strange particles? When comparing the rates of lambda and neutron β-decays, after correcting for the different energy releases, it is found that

$$\frac{\text{rate}\,(\Lambda \rightarrow p + e^- + \bar{\nu}_e)}{\text{rate}\,(n \rightarrow p + e^- + \bar{\nu}_e)} = 0.053.$$

Determine a value for the Cabibbo angle.

[7 marks]

SEE NEXT PAGE

SECTION B – Answer TWO questions

2.) a) Briefly discuss why three generations of elementary particles appear to required. State one piece of evidence which indicates that there is not a fourth generation.

[4 marks]

b) Write down the lepton and quark doublets of each of the generations. Discuss how the doublets may be mixed.

[6 marks]

c) State why the quark structures of some baryons caused an initial problem with the quark model, and describe how this problem was overcome by the introduction of the quantum number *colour*.

[4 marks]

d) The strong interaction between quarks is considered to take place through the exchange of gluons. Which quantum number is exchanged? Describe qualitatively an important difference between this view of the strong interaction and the electromagnetic interaction. Briefly discuss the implications of this difference.

[6 marks]

e) *Estimate* the mass of the strange quark given that $m_{\mathrm{N}} \approx 940\,\mathrm{MeV/c^2}$ (N is a nucleon), $m_{\Lambda} \approx 1115\,\mathrm{MeV/c^2}$, $m_{\Sigma} \approx 1190\,\mathrm{MeV/c^2}$ and $m_{\Xi} \approx 1320\,\mathrm{MeV/c^2}$. Repeat the estimate given that $m_{\Delta} \approx 1230\,\mathrm{MeV/c^2}$, $m_{\Sigma*} \approx 1385\,\mathrm{MeV/c^2}$, $m_{\Xi*} \approx 1530\,\mathrm{MeV/c^2}$ and $m_{\Omega} \approx 1670\,\mathrm{MeV/c^2}$. State any assumptions you have made and comment on any differences between the two estimates.

[10 marks]

3.) a) Define what is meant by (i) the four-momentum of a particle, (ii) a conserved quantity and (iii) an invariant quantity. Give one example of a conserved quantity which is not invariant, one of an invariant quantity which is not conserved and one of a quantity which is both conserved and invariant.

[9 marks]

b) Define the system of natural units. Hence obtain values, in SI units, for the natural units of length, energy and time.

[6 marks]

c) Show that when a particle of mass m collides with a stationary particle of mass M, the minimum energy required to produce a state of mass M^* is, in natural units,

$$E_{\text{threshold}} = \frac{M^{*2} - M^2 - m^2}{2M}.$$

[6 marks]

d) Determine the threshold energy for producing a top quark and an anti-top quark when beams of electrons and positrons of equal energy are made to collide head on.

[3 marks]

e) If a beam of electrons, of the same energy as in part d), struck a stationary proton target, what is the maximum mass particle that could be produced? Comment on your answer.

[6 marks]

4.) a) Write down four properties of SU(3) multiplets.

[4 marks]

b) Draw and fully label the fundamental SU(3) triplet, and its conjugate, of the three lightest quarks.

[6 marks]

c) On the fundamental triplet diagram, indicate the effect of the SU(3) operators $\mathbf{I}_{\pm}$, $\mathbf{U}_{\pm}$ and $\mathbf{V}_{\pm}$. Show that these operators satisfy the closure condition necessary for them to be members of a group.

[8 marks]

d) Imagine that the quark model allowed the existence of particles containing two quarks and one antiquark. Determine the SU(3) multiplets which the particles would be grouped in, given that a quark and an antiquark result in an octet and the singlet.

[8 marks]

e) Determine the SU(2) subsets of the multiplet of highest weight from part (d).

[4 marks]

5.) a) Recently there have been several important experimental results in particle physics. These include (i) the discovery of neutrino oscillations, (ii) the possible observation of the quark-gluon plasma, (iii) the direct observation of tau neutrino interactions and (iv) the possible discovery of the Higgs Boson. Make brief notes on each of the experiments.

[20 marks]

b) Discuss the relative significance of each of these results and select the one which you believe to be the most significant, giving reasons for your choice.

[10 marks]

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