King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP2621 Astrophysics

Summer 2006

Time allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A, and no more than TWO questions from SECTION B. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	$=$	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$=$	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$=$	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	$=$	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	$=$	1.602×10^{-19}	C
Electron rest mass	m_{e}	$=$	9.109×10^{-31}	kg
Unified atomic mass unit	m_{u}	$=$	1.661×10^{-27}	kg
Proton rest mass	m_{p}	$=$	1.673×10^{-27}	kg
Neutron rest mass	m_{n}	$=$	1.675×10^{-27}	kg
Planck constant	h	$=$	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_{B}	$=$	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	$=$	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	$=$	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_{A}	$=$	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		$=$	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	$=$	1.013×10^{5}	$\mathrm{N\,m^{-2}}$
Electron volt	$1\,\mathrm{eV}$	$=$	1.602×10^{-19}	J
Mass of the Sun	$M_\odot$	$=$	1.989×10^{30}	kg
Radius of the Sun	$R_\odot$	$=$	6.960×10^{8}	m
Luminosity of the Sun	$L_\odot$	$=$	3.827×10^{26}	W

SECTION A – Answer all SIX parts of this section

1.1) The Wien displacement law is

$$\lambda_{\mathrm{max}} = \frac{2.898 \times 10^{-3}}{T}.$$

If the wavelength of the maximum emission, λ_{max}, of the supergiant star Rigel is 223 nm and its luminosity is $7.5 \times 10^4\, L_\odot$, what are its effective temperature, T, and radius, R, in units of $R_\odot$?

[7 marks]

1.2) The Boltzmann equation gives the ratio of the number, N, of atoms with energy E_b to the number with energy E_a. It can be written as

$$\frac{N(E_b)}{N(E_a)} = \frac{g_b}{g_a} \exp\left(-\frac{E_b - E_a}{k_{\mathrm{B}}T}\right),$$

where all the symbols have their usual meanings. Consider a gas of *neutral* hydrogen atoms. Given that the ionisation energy of hydrogen is 13.6 eV, use the Boltzmann equation to find the temperature at which there are 5×10^3 as many atoms with electrons in the ground state as there are with electrons in the first excited state.

[7 marks]

1.3) Three of the equations of stellar structure are

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)\rho(r)}{r^2} \tag{1}$$

$$\frac{\mathrm{d}M(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r) \tag{2}$$

$$\frac{\mathrm{d}L(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r)\epsilon(r). \tag{3}$$

State what each of the equations represents and also what each of the symbols within them stands for (excluding any constants).

[7 marks]

1.4) From stellar evolution theories we know that a star's mass, M, and luminosity, L, are related by $L \propto M^{3.5}$. Estimate the main-sequence timescale, τ_{ms}, of a $2\,M_\odot$ star, using the fact that for the Sun, $\tau_{\mathrm{ms}} = 1.5 \times 10^{10}$ yrs. You may assume that $\tau_{\mathrm{ms}} \propto M/L$.

[7 marks]

1.5) Explain what is meant by *helium flash* and *degenerate electron pressure*, clearly stating why the condition of degeneracy is necessary for the former to occur. State what mass stars experience a helium flash event, and at what stage of their evolution.

[7 marks]

1.6) Describe what is meant by a *neutron star*. Calculate the mass in $M_\odot$ and the surface gravity, $g = GM/R^2$, in SI units, of a neutron star with a diameter of 20 km and a density of $10^{18}\,\mathrm{kg\,m^{-3}}$. Compare your result to the surface gravity of the Sun.

[7 marks]

SECTION B – Answer TWO questions

2a) i) Define *fully* each of the symbols in the equation of radiative transfer

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = -I_\lambda + S_\lambda.$$

[7 marks]

ii) Assuming S_λ to be constant, show that the solution of this equation, with boundary condition $I_\lambda(\tau = 0) = I_{\lambda_0}$, is

$$I_\lambda = I_{\lambda_0} e^{-\tau_\lambda} + S_\lambda(1 - e^{-\tau_\lambda}).$$

[8 marks]

b) Consider a star of radius, R, made up of thin spherical shells of gas, of mass, $\mathrm{d}M(r)$. The total gravitational potential energy, E_G, and total thermal energy, E_{th}, of the star are given by

$$E_G = -\int_0^R GM(r) 4\pi r \rho \, \mathrm{d}r \qquad \text{and} \qquad E_{\mathrm{th}} = \int_0^R \frac{3}{2} P_{\mathrm{th}} 4\pi r^2 \, \mathrm{d}r,$$

where ρ is the density and P_{th} is the thermal pressure. Starting with the following stellar structure equation

$$\frac{\mathrm{d}P_{\mathrm{th}}}{\mathrm{d}r} = -\frac{GM(r)\rho}{r^2},$$

show that the virial theorem is

$$E_{\mathrm{th}} = -\frac{1}{2} E_G.$$

[8 marks]

c) Another of the equations of stellar structure is

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\chi(r)\rho(r)L(r)}{64\pi\sigma r^2 T^3}.$$

State what each of the symbols in this equation stands for (excluding any constants) and also what physical process the equation represents. Describe the conditions under which the equation becomes invalid.

[7 marks]

3a) Explain, with the aid of a diagram, how parallax can be used to measure a star's distance. What is the distance (in parsecs) of a star with a parallax of 0.125 arcseconds?

[6 marks]

b) The apparent magnitudes, m, of stars in a cluster are measured and plotted in a Hertzsprung-Russell (H-R) diagram. This plot is compared with an absolute magnitude, M, H-R diagram of nearby stars whose distances are known. The main sequence of stars in the cluster is found to be 12 magnitudes fainter than that of the main sequence of nearby stars. How far away is the cluster? What assumption has been made about the stars in the cluster?

[5 marks]

c) Explain how an H-R diagram can be used to estimate the age of a star cluster.

[4 marks]

d) Describe the post-main-sequence evolution of a $1\,M_{\odot}$ star up to the formation of a planetary nebula, clearly labelling all stages on an H-R diagram. In particular, mark on the diagram the approximate locations of the first dredge-up, the helium flash, the asymptotic giant branch, the onset of thermal pulses and the planetary nebula ejection.

[15 marks]

4a) i) Explain what is meant by a *white dwarf*.

[3 marks]

ii) Describe the three types of white dwarf that may be formed, clearly stating what main physical stellar property is responsible for the formation of each.

[6 marks]

b) i) White dwarfs are supported against gravity by degenerate gas pressure, P. This is proportional to a power of the gas density, ρ, according to the following, approximate relation

$$P \propto \rho^{5/3}.$$

Assuming that in hydrostatic equilibrium, P, the mass, M, and radius, R, of a white dwarf are related as follows

$$P \propto \frac{M^2}{R^4},$$

derive a relation between the radius of a white dwarf and its mass.

[6 marks]

ii) What is the significance of this result?

[2 marks]

c) i) Give a definition of the *Chandrasekhar limit*, $M_{\rm ch}$.

[2 marks]

ii) A simple treatment for a relativistic white dwarf gives

$$M_{\rm ch} = \frac{5.87}{\mu^2} M_{\odot},$$

where μ is the number of nucleons per electron. Calculate a value for $M_{\rm ch}$ using μ for iron.

[3 marks]

iii) A more accurate treatment would give a Chandrasekhar limit of $1.44\,M_{\odot}$. Describe what happens in cases when the Chandrasekhar limiting mass is exceeded.

[4 marks]

iv) Using simple arguments derive an expression for the Schwarzschild radius, $R_{\rm sc}$, of a black hole of mass, M.

[4 marks]