King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/2621 Astrophysics

January 2003

Time allowed: THREE Hours

Candidates should answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

**You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.**

TURN OVER WHEN INSTRUCTED

Electron charge	e =	1.602×10^{-19} C
Electron mass	m_e =	9.109×10^{-31} kg
Hydrogen mass	m_H =	1.674×10^{-27} kg
Boltzmann constant	k_B =	1.381×10^{-23} J K^{-1}
Planck constant	h =	6.626×10^{-34} J s
Gravitational constant	G =	6.672×10^{-11} N m^2 kg^{-2}
Permittivity of free space	ϵ_0 =	8.854×10^{-12} C^2 N^{-1} m^{-2}
Mass of the Sun	$M_\odot$ =	1.989×10^{30} kg
Radius of the Sun	$R_\odot$ =	6.960×10^{8} m

SECTION A – Answer SIX parts of this section

1.1) The star Sirius is a binary system. Sirius A is the brighter star and has an apparent visual magnitude of -1.46. The ratio of the luminosities of the stars is 9.6×10^3. What is the apparent visual magnitude of Sirius B?

[7 marks]

1.2) State what is meant by the 'effective temperature' of a star. Explain, with the aid of an equation, why two stars with the same effective temperature but with different luminosities must have different sizes.

[7 marks]

1.3) The luminosity of a star is determined to be related to its mass according to the following relation

$$L \propto M^{3.5}.$$

Given that the Sun is expected to have a main sequence lifetime of 1.5×10^{10} yr, estimate the main sequence lifetime of a 3 $M_\odot$ star.

[7 marks]

1.4) Sketch the interior zones of a medium mass (1 $M_\odot$) star and of a low mass (0.1 $M_\odot$) star, as inferred from computer solutions to the stellar structure equations. Show the approximate location of any convective or radiative zones and the relative core radius. Briefly describe the principal core nuclear burning process for the two stars.

[7 marks]

1.5) The solution of the equation of radiative transfer for a beam of light passing through a volume of gas with optical depth τ_λ is given by

$$I_\lambda = I_{\lambda 0} \mathrm{e}^{-\tau_\lambda} + S_\lambda (1 - \mathrm{e}^{-\tau_\lambda}),$$

where $I_{\lambda 0}$ is the incident intensity and S_λ is the source function. Describe the physical significance of all the terms in this equation.

[7 marks]

1.6) Describe what is meant by Local Thermodynamic Equilibrium (LTE). Give an example of a region in a star where LTE may be a bad assumption and explain why.

[7 marks]

1.7) Describe how Hertzsprung-Russell diagrams can be used to estimate the ages and distances of stellar clusters.

[7 marks]

1.8) The following equation is used in modelling stellar structure:

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{16\sigma} \frac{\kappa}{T^3} \frac{L_r}{4\pi r^2}.$$

Define all the terms in this equation and state the physical process which it represents. Describe the circumstances under which this equation becomes inapplicable.

[7 marks]

SECTION B – Answer TWO questions

2) Sketch and label a Hertzsprung-Russell diagram. Indicate the main sequence, and the approximate positions of red giant, red supergiant and white dwarf stars. Include the position of the Sun on the diagram and draw a line showing its future evolution from the main sequence to the white dwarf stage. Indicate on the line the red giant branch, helium flash point, asymptotic giant branch and thermal pulses.

[12 marks]

Give a detailed account of the evolution of a 1 $M_\odot$ star from the main sequence to the helium flash point.

[18 marks]

3) By considering the pressure and gravitational force on a small element of area $\mathrm{d}A$ and thickness $\mathrm{d}r$ at a distance r from the centre of a sphere of gas, derive the equation of hydrostatic support,

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{GM_r\rho(r)}{r^2},$$

where M_r is the mass enclosed at a radius r, $P(r)$ is the pressure and $\rho(r)$ is the mass density.

By considering the mass enclosed in a thin shell at distance r from the centre of the star show that

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2\rho(r).$$

[12 marks]

Suppose that a star is modelled as having a density that decreases linearly from the centre to the surface,

$$\rho(r) = \rho_c\left(1 - \frac{r}{R}\right),$$

where ρ_c is the central density and R is the stellar radius.

Show that

$$M_r = \frac{4\pi}{3}\rho_c r^3\left(1 - \frac{3r}{4R}\right),$$

and thus that

$$\rho_c = \frac{3M}{\pi R^3},$$

where M is the mass of the star.

[6 marks]

Use the equation of hydrostatic support and the expressions for M_r and ρ_c to show that the central pressure is

$$P_\mathrm{c} = \frac{5}{4\pi}\frac{GM^2}{R^4},$$

where M and R are the mass and radius of the star. Hence, using the ideal gas law $P = \rho k_\mathrm{B}T/\mu m_\mathrm{H}$, show that the central temperature is

$$T_\mathrm{c} = \frac{5}{12}\frac{\mu m_H}{k_\mathrm{B}}\frac{GM}{R}.$$

[8 marks]

Using $\mu = 0.6$, calculate values for the Sun's central pressure and temperature in this model.

[4 marks]

4) Summarise the arguments that led to nuclear fusion being identified as the principal power source in the Sun.

[6 marks]

Show that the classical temperature required for two repulsing particles (with charges $z_1 e$ and $z_2 e$) to achieve a minimum separation r is

$$T_{\text{classical}} = \frac{2}{3}\frac{z_1 z_2 e^2}{4\pi\epsilon_o k_{\text{B}} r}.$$

In order for two protons to overcome the Coulomb barrier they must approach to within 10^{-15} m of each other. Calculate the classical temperature required for this occur.

[6 marks]

Describe, with the aid of a sketch, how it is possible for fusion to occur at the temperature of the core of the Sun, 10^7 K.

[6 marks]

Describe the pp chain, CNO cycle and triple-α nuclear reaction sequences. Do not give details of individual reactions but give the initial and final products and any important catalyst reactants. State the temperatures at which each of these sequences are important and give examples of the types of star in which they may found.

[12 marks]

5) The Boltzmann equation is

$$\frac{N(E_\mathrm{a})}{N(E_\mathrm{b})} = \frac{g_\mathrm{a}}{g_\mathrm{b}} \exp\left[-\frac{(E_\mathrm{a} - E_\mathrm{b})}{k_\mathrm{B}T}\right].$$

The Saha equation is

$$\frac{N_{i+1}}{N_i} = \frac{2k_\mathrm{B}T}{P_\mathrm{e}}\frac{Z_{i+1}}{Z_i}\left(\frac{2\pi m_\mathrm{e}k_\mathrm{B}T}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_i}{k_\mathrm{B}T}\right).$$

Define all the terms in these equations (apart from the physical constants).

[5 marks]

Consider a stellar atmosphere composed of pure hydrogen at a temperature of 12,000 K and a pressure of 20 N m^{-2}. Use the Boltzmann equation to find the ratio of neutral hydrogen atoms which are in the $n = 2$ state relative to those that are in the $n = 1$ state. The energy of the n^th state of hydrogen is $-13.6/n^2$ ev.

[7 marks]

Use the Saha equation to calculate the ratio of ionized to neutral hydrogen atoms in this stellar atmosphere. For hydrogen, $Z_\mathrm{I} = 2$ and $Z_\mathrm{II} = 1$.

[7 marks]

Use these results to calculate the fraction of all hydrogen atoms that are in the $n = 2$ state and thus able to generate a Balmer absorption line. The number of atoms in the $n = 3$ and higher states can be assumed to be negligible.

[6 marks]

Explain why the strength of the hydrogen Balmer absorption lines is a maximum at approximately 10,000 K for main sequence stars.

[5 marks]