King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/2621 Astrophysics

Summer 2002

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

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Electron charge	e =	1.602×10^{-19} C
Atomic mass unit	$\mathrm{m_u}$ =	1.662×10^{-27} kg
Atomic mass of a proton	m_p =	$1.0078\ \mathrm{m_u}$
Atomic mass of ^{4}He	m_α =	$4.0026\ \mathrm{m_u}$
Speed of light in free space	c =	$2.998 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Boltzmann constant	k =	$1.381 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Gravitational constant	G =	$6.672 \times 10^{-11}\ \mathrm{N\,m^2\ kg^{-2}}$
Mass of the sun	$M_\odot$ =	1.989×10^{30} kg
Radius of the sun	$R_\odot$ =	6.96×10^{8} m
Luminosity of the sun	$L_\odot$ =	$3.85 \times 10^{26}\ \mathrm{J\,s^{-1}}$
Number of seconds in a year	1 yr =	3.16×10^{7} s

SECTION A – Answer SIX parts of this section

1.1) The star Sirius is a binary system. Sirius A has an apparent visual magnitude of -1.46 while Sirius B has an apparent visual magnitude of 8.5. Calculate the ratio of the luminosities of the stars.

Given that the absolute magnitude of Sirius A is 1.45, calculate the distance to the Sirius system.

[7 marks]

1.2) The radiation from a protostar of radius 10^6 km is absorbed by a surrounding spherical dust cloud of radius 4×10^8 km. If all the radiation from the protostar is re-emitted by the gas cloud and the cloud is observed to have a temperature of 500 K, what is the effective temperature of the protostar?

[7 marks]

1.3) Give the definition of optical depth and briefly describe two sources of opacity in stellar atmospheres.

[7 marks]

1.4) By using a semi-classical argument, show that the Schwarzchild radius for a black hole is $R_\mathrm{sch} = 2GM/c^2$. Calculate R_sch for a black hole of mass $2M_\odot$.

[7 marks]

1.5) Describe *two* of the PP chain, CNO cycle and Triple-α nuclear reaction sequences. Do not give details of individual reactions but give the initial and final products and any important catalyst reactants. State the temperatures at which each of these sequences are important and give an example of the types of star in which they may found.

[7 marks]

1.6) Calculate the total nuclear energy available in a $5M_\odot$ star if 10% of the star's mass is converted from H to He. Hence, estimate the main sequence lifetime (in years) of this star if it has a luminosity of $600L_\odot$.

[7 marks]

1.7) The Boltzmann equation is

$$\frac{N(E_\mathrm{a})}{N(E_\mathrm{b})} = \frac{g_\mathrm{a}}{g_\mathrm{b}} \exp\left[-\frac{(E_\mathrm{a} - E_\mathrm{b})}{kT}\right],$$

where the symbols have their usual meanings. For a gas of neutral atomic hydrogen, at a temperature of 10^4 K, calculate the ratio of the number of atoms which have an electron in the first excited state to those which have an electron in the ground state. The energy of the ground state of hydrogen is -13.6 eV.

[7 marks]

1.8) An $8M_\odot$ star has evolved to the point where the inner core consists mainly of ${}^{56}_{26}\mathrm{Fe}$. List the sequence of events that give rise to a type II supernova as the core of this star collapses.

[7 marks]

SECTION B – Answer TWO questions

2) Sketch and label a Hertsprung-Russell (HR) diagram. Indicate the main sequence, and the approximate positions of red giant, red supergiant and white dwarf stars. Include the position of the sun on the diagram and draw a line showing its future evolution from the main sequence to the white dwarf stage. Indicate on the line the red giant branch, helium flash point, asymptotic giant branch and thermal pulses.

[20 marks]

Describe how HR diagrams can be used to estimate the ages of star clusters.

[6 marks]

The apparent magnitudes of stars from a cluster are plotted on an HR diagram. Also on the diagram are plotted the absolute magnitudes of stars from a theoretical zero age main sequence (ZAMS). The main sequence of the cluster is parallel to, and separated vertically from, the ZAMS and is fainter by 8.4 magnitudes. How far away is the cluster?

[4 marks]

3) By considering the pressure and gravitational force on a small element of area $\mathrm{d}A$ and thickness $\mathrm{d}r$ at a distance r from the centre of a sphere of gas, derive the equation of hydrostatic support,

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{GM_r\rho(r)}{r^2},$$

where M_r is the mass enclosed at a radius r, $P(r)$ is the pressure and $\rho(r)$ is the mass density.

By considering the mass enclosed in a thin shell at distance r from the centre of the star show that

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2 \rho(r).$$

[10 marks]

Suppose that a star is modelled as having a constant density. Use the equations above to show that the central pressure is

$$P_\mathrm{c} = \frac{3}{8\pi}\frac{GM^2}{R^4},$$

where M and R are the mass and radius of the star. Hence, use the ideal gas law $P = \rho kT/\mu m_u$ to show that the central temperature is then

$$T_\mathrm{c} = \frac{\mu m_u}{2k}\frac{GM}{R}.$$

[12 marks]

Using $\mu = 0.6$, calculate numerical values for the sun's central pressure and temperature with this model. Why are these underestimates of the true central pressure and temperature?

[8 marks]

4) The equation of radiative transfer relating the change of intensity at a given wavelength, I_λ, with optical depth τ_λ can be written as

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = -I_\lambda + S_\lambda,$$

where S_λ is the source function at this wavelength and optical depth.

Taking S_λ to be constant, show that the solution of the equation of transfer with the boundary condition $I_\lambda(\tau = 0) = I_{\lambda 0}$ is given by

$$I_\lambda = I_{\lambda 0} e^{-\tau_\lambda} + S_\lambda(1 - e^{-\tau_\lambda}).$$

[15 marks]

What does this equation approximate to when $I_{\lambda 0} \neq 0$ and the stellar material is (i) optically thin, and (ii) optically thick? What are the possible emergent spectra that result from these cases? Hence explain why, in a stellar spectrum, absorption lines on a continuum background are usually seen.

[15 marks]

5) The following equations are used to model stellar structures:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r}, \tag{1}$$

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2\rho, \tag{2}$$

$$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2\rho\epsilon, \tag{3}$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{16\sigma}\frac{\kappa}{T^3}\frac{L_r}{4\pi r^2}, \tag{4}$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = (1-\frac{1}{\gamma})\frac{T}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r}, \tag{5}$$

$$P = \frac{\rho kT}{\mu m_u}. \tag{6}$$

Equations 1, 2 and 3 are, respectively, the equations of hydrostatic equilibrium, mass conservation and energy conservation. Equation 6 is the equation of state.

State what equations 4 and 5 represent and describe the circumstances under which each are applicable.

[6 marks]

What are the boundary conditions required to solve these equations? Give two further requirements for a physical solution. State the Vogt-Russell theorem for stellar evolution.

[12 marks]

Sketch the differing interiors of high ($50M\odot$), medium ($1M\odot$) and low ($0.1M\odot$) mass main sequence stars as inferred from computer solutions to the stellar structure equations. Show the approximate location of any convective or radiative zones and state the principal core nuclear burning process for each star.

[12 marks]