King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/2481 Electromagnetism and Optics 1 – RESIT Paper

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) When a dielectric substance is subjected to an electric field of strength $\mathbf{E}$ it becomes *polarised*. Discuss the meaning of this term and define the polarisation $\mathbf{P}$ and electrical susceptibility χ of the dielectric.

[7 marks]

1.2) Write a brief *qualitative* account of the forces that are present in a capacitor that contains a material dielectric. A mathematical analysis is *not* required.

[7 marks]

1.3) State Ampère's circuital law.
Consider a long straight wire *in free space* with a circular cross section of radius a. If I is the current in the wire and the permeability of the wire material is μ,

a) draw a diagram that illustrates the magnetic field strength $\mathbf{H}$ both inside and outside the wire.

b) Obtain an expression for the magnetic induction $\mathbf{B}$ at a distance r from the axis of the wire where $r \leq a$.

[7 marks]

1.4) State Maxwell's equations for the electromagnetic field and give the S.I. units of all the quantities involved.

[7 marks]

1.5) The plane-wave solution of the wave equation for an electromagnetic field within a medium of permittivity ε and permeability μ is of the form

$$\mathbf{F} = \mathbf{F}_1\,(x - vt) + \mathbf{F}_2\,(x + vt).$$

Discuss the physical nature of this solution, stating the relation between the parameters v, ε and μ.

[7 marks]

1.6) Briefly distinguish between the conditions needed to observe Fraunhofer and Fresnel diffraction.

Comment qualitatively on how the diffracted intensity at a fixed distance from a long narrow slit would change as the width of the slit is slowly increased.

[7 marks]

1.7) An aperture is illuminated by a monochromatic plane wave. Explain why the diffracted intensity in the far field approximation is unaffected by a displacement of the diffracting aperture in a plane perpendicular to the direction of the illumination.

[7 marks]

1.8) Write down an expression for the chromatic resolving power of a regular linear diffraction grating with N slits, defining all the symbols used.

Hence determine the minimum number of slits that would be needed for the grating to just resolve the two yellow sodium D lines, with wavelengths 588.995 nm and 589.592 nm, using the 5th-order principal maxima.

[7 marks]

SECTION B – Answer TWO questions

2) Explain the meaning of the term *polarisation charge* as used in electrostatics.

[2 marks]

When a dielectric is placed in an electric field of strength $\mathbf{E}$, polarisation charges appear on its surface and throughout its volume. If $\mathbf{P}$ is the electrical polarisation, prove that

a) the surface density of polarisation charge is $\sigma_P = \mathbf{P} \cdot \mathbf{n}$, where $\mathbf{n}$ is the unit normal to the surface.

b) the volume density of polarisation charge is $\rho_P = -\nabla \cdot \mathbf{P}$.

[10 marks]

A capacitor is constructed using two concentric metal cylinders, the inner cylinder having a radius of 1 cm and the outer cylinder a radius of 3 cm. The capacitor is completed by threading a dielectric tube with an inner radius of 1 cm and an outer radius of 2 cm onto the central electrode. The relative permittivity of this dielectric is $\varepsilon_r = 2.0$.
Prove that the capacitance per meter of length of this capacitor is

$$C = \frac{2\pi\varepsilon_0}{\log_e(3/\sqrt{2})}.$$

[10 marks]

If the charge on the inner cylinder is 1 nC per meter of length, and the outer electrode is earthed,

a) calculate the polarisation charge densities on the inner and outer surfaces of the dielectric.

b) Show that the potential difference between the capacitor plates is 13.5 V.

[8 marks]

3) Write a brief account of the phenomenon of *ferromagnetism*.

[12 marks]

Discuss the meaning of the term *Ampèrian current*. Your answer should include a statement of the relations between the magnetisation $\mathbf{M}$ of the medium and the surface and volume Ampèrian current densities $\mathbf{J}_A$ and $\jmath_A$. A mathematical derivation of these results is *not* required.

[8 marks]

A permanently magnetised sphere of radius a is located in free space. The magnetisation $\mathbf{M}$ of the sphere is uniform and points in the z-direction. Using spherical polar coordinates (r, ϑ, φ), where ϑ is measured from the positive z-direction, obtain formulas for the Ampèrian surface and volume current distributions.
Draw a sketch to illustrate the surface distribution.

[6 marks]

If $a = 5\,\mathrm{cm}$ and $M = 10^6\,\mathrm{Am}^{-1}$, prove that the Ampèrian current I_A flowing on the surface of the sphere is equal to 10^5 A.

[4 marks]

4 (a) Derive the *equation of continuity*.

[6 marks]

Assuming the formula for the velocity of propagation of an electromagnetic wave in a medium of relative permittivity ε_r and relative permeability μ_r, show that the refractive index of the medium n is given by $n = \sqrt{\mu_r \varepsilon_r}$.

[4 marks]

Discuss the problem encountered in verifying this result experimentally.

[5 marks]

(b) By considering two objects with complex amplitude transmission functions $f(x)$ and $1 - f(x, y)$, deduce Babinet's principle, as it applies to the far-field diffraction patterns from complementary apertures.

[6 marks]

A particular object is illuminated with a monochromatic plane wave of wavelength λ and the diffracted intensity in the far field is found to have the form

$$I(\theta) = I_0 \left(\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right)^2 \left(\frac{\sin(5\pi d \sin\theta/\lambda)}{\sin(\pi d \sin\theta/\lambda)} \right)^2$$

Without carrying out a detailed calculation, explain clearly the deductions you can make about the general form of the diffracting object.

[5 marks]

Describe the effect on the diffracted intensity given above if the parameters a and d satisfy the condition $d = 4a$.

[4 marks]

5) Discuss briefly how the convolution theorem for Fourier transforms can be used when dealing with the problem of Fraunhofer diffraction from arrays of identical apertures.

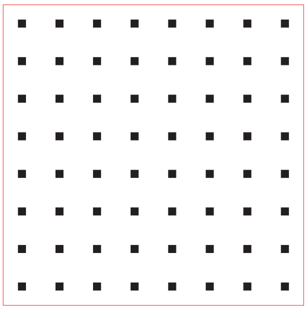
[6 marks]

Derive an expression for the far-field diffraction pattern obtained from a set of N identical apertures that are positioned randomly in a plane, assuming that N is a very large number, and that none of the apertures overlap.

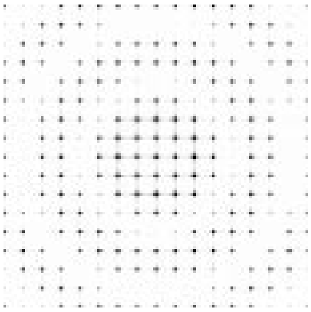
[9 marks]

Figure 5.i shows five objects from which the Fraunhofer diffraction pattern is to be obtained. Figure 5.ii shows five possible Fraunhofer diffraction patterns. Identify which of the diffraction patterns should be associated with each of the diffracting objects shown, briefly stating a reason for each of your choices.

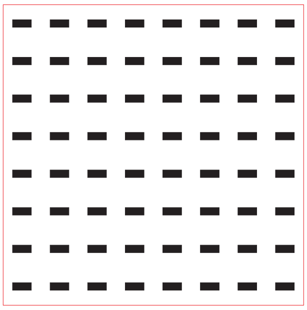
[15 marks]



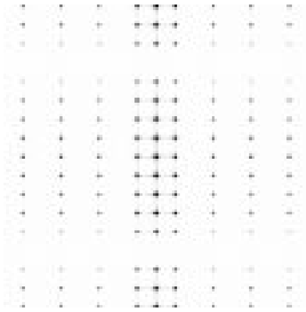
5.i Object 1



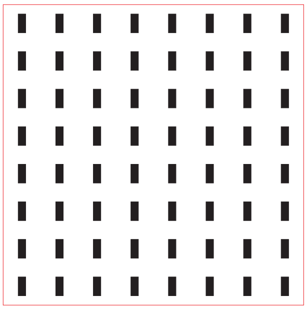
5.ii Diffraction pattern A



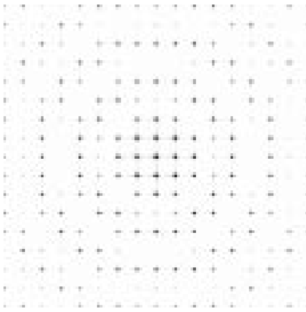
5.i Object 2



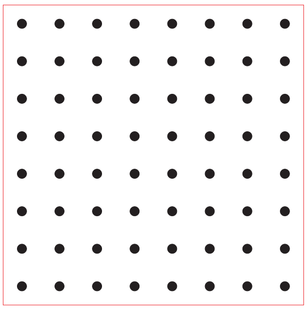
5.ii Diffraction pattern B



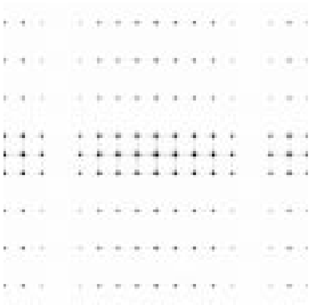
5.i Object 3



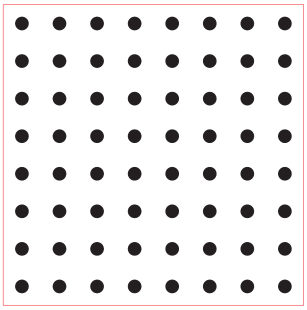
5.ii Diffraction pattern C



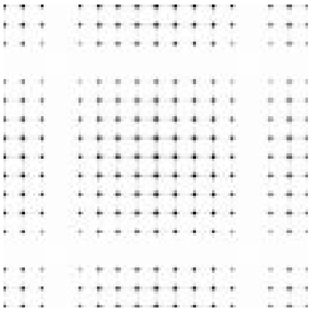
5.i Object 4



5.ii Diffraction pattern D



5.i Object 5



5.ii Diffraction pattern E