King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/2470 PRINCIPLES OF THERMAL PHYSICS

Summer 1998

Time allowed: THREE Hours

Candidates should answer SIX parts of SECTION A,
and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Throughout this paper, P denotes the pressure, T the thermodynamic temperature, V the volume, v the molar volume and R the universal gas constant.

SECTION A – Answer SIX parts of this section

1.1) Four moles of an ideal gas, at an initial temperature T_o, are compressed quasi-statically and isobarically such that the volume is halved. Show that the work done is $2RT_o$.

[7 marks]

1.2) Define the internal energy U of a thermodynamic system. Deduce the expression $\mathrm{d}U = đQ + đW$, defining all the terms carefully.

[7 marks]

1.3) State the Principle of Equipartition of Energy and hence show that the molar heat capacity C_P of a monatomic ideal gas is $\frac{5}{2}R$.

(Assume $C_P - C_V = R$, where C_P and C_V are the molar heat capacities at constant pressure and volume, respectively.)

[7 marks]

1.4) Explain why the following statements are **incorrect**:

(a) Cooling a specimen reduces its entropy and hence the Principle of Increasing Entropy is violated;

(b) The expression $\mathrm{d}S = \frac{đQ}{T}$ is valid for a reversible process and cannot, therefore, be used to calculate the entropy change of a system as a result of an irreversible process.

[7 marks]

1.5) Calculate the change of entropy of 100 g of lead during the following processes:

(a) Cooling the lead from 100 °C to 20 °C;

(b) Melting the lead which is initially at its melting point, 327 °C.

(Assume that the specific heat capacity of lead in the range 20 to 100 °C is equal to 127 $\mathrm{J\,kg^{-1}K^{-1}}$ and the latent heat of fusion is $2.5 \times 10^4\ \mathrm{J\,kg^{-1}}$.)

[7 marks]

1.6) The *energy equation* for any system with the state variables P, V and T is

$$\left(\frac{\partial U}{\partial V}\right)T = T\left(\frac{\partial P}{\partial T}\right)V - P\,,$$

where U is the internal energy. Using this equation, prove that the internal energy of an ideal gas is a function of T only. Hence, show that there is no temperature change accompanying the free expansion of an ideal gas.

[7 marks]

1.7) Derive the equation,

$$T\mathrm{d}S = C_P\mathrm{d}T - T\left(\frac{\partial V}{\partial T}\right)_P \mathrm{d}P\,.$$

[7 marks]

1.8) The Earth orbits the Sun at a distance of 1.49×10^{11} m. At this distance the radiant solar flux is $1.4 \times 10^3\,\mathrm{J\,m^{-2}\,s^{-1}}$. Calculate the temperature of the Sun assuming it to be a black body of diameter 1.39×10^9 m. The value of Stefan's constant is $\sigma = 56.7\,\mathrm{nW\,m^{-2}K^{-4}}$.

[7 marks]

SECTION B – Answer TWO questions

2) Define the terms *critical temperature* and *critical volume.* Show how they may be obtained from an equation of state.

[8 marks]

A mole of gas is contained in a volume of 1 litre, and it is found that its pressure is 95% of that predicted by the ideal gas theory for the same temperature and volume. Calculate the temperature of the gas, given that its critical volume is $75 \times 10^{-6}\,\mathrm{m^3mol^{-1}}$ and its critical temperature is 160 K.

(Assume that the gas obeys Van der Waals' equation of state

$$(P + \frac{a}{v^2})(v - b) = RT\,,$$

where the symbols have their usual meanings.)

3) By calculating the change in entropy for a complete cycle of a Carnot heat engine, show that its efficiency η_c is given by

$$\eta_c = 1 - \frac{T_2}{T_1},$$

where T_1 and T_2 are the temperatures of the hot and cold reservoirs, respectively.

[12 marks]

A heat pump, operating a Carnot cycle in reverse, is used to maintain a temperature of 17 °C inside a building when the temperature outside is 2 °C. Under these conditions, the building loses heat at the rate of 10^4 J s^{-1} and the heat pump's efficiency is half that of a Carnot heat pump.

(a) Calculate the power required to run the pump.

[12 marks]

(b) Compare the cost of heating the building using the above pump with that of using electric fires to maintain the same temperature, assuming that the cost of electrical energy is 7p/kWh.

[6 marks]

4) Define the Helmholtz function F of a thermodynamic system in equilibrium.

[3 marks]

Explain what is meant by 'F is a state function'.

[4 marks]

Derive relations which enable the pressure and entropy of the system to be derived from F.

[6 marks]

The Helmholz function of a fluid is given by

$$F = AT(1 - \ell\text{n}T) - BT\,\ell\text{n}V + C$$

where A, B, C are constants.

(a) Derive expressions for the pressure, entropy and internal energy of the system.

[8 marks]

(b) Identify the type of fluid, and hence show $A + B$ is the thermal capacity of the fluid at constant pressure.

[9 marks]

5) Write an essay describing (a) how liquid helium may be produced and (b) how temperatures around 0.01K may be achieved.

[30 marks]