King's College London

University of London

B.Sc. EXAMINATION

CP/2380 Electromagnetism

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A — Answer SIX parts of this section

1.1) Describe briefly the meaning of the term *dielectric* and give three examples of dielectric materials.
[7 marks]

1.2) When a dielectric substance is subjected to an electric field of strength $\boldsymbol{E}$ it becomes *polarised*. Discuss the meaning of this term and define the polarisation $\boldsymbol{P}$ and electrical susceptibility χ of the dielectric.
[7 marks]

1.3) Write a brief *qualitative* account of the forces that are present in a capacitor that contains a material dielectric. A mathematical analysis is *not* required.
[7 marks]

1.4) State Ampère's circuital law.
Consider a long straight wire *in free space* with a circular cross section of radius a. If I is the current in the wire and the permeability of the wire material is μ;
a) draw a diagram that illustrates the magnetic field strength $\boldsymbol{H}$ both inside and outside the wire.
b) Obtain an expression for the magnetic induction $\boldsymbol{B}$ at a distance r from the axis of the wire where $r \leq a$.
[7 marks]

1.5) Give a brief account of the phenomenon of *diamagnetism*.
[7 marks]

1.6) State the complex form of Maxwell's equations for an electromagnetic field propagating in a linear medium of relative permittivity ϵ_r, relative permeability μ_r and electrical conductivity σ defining all of the symbols used.
[7 marks]

1.7) The plane-wave solution of the wave equation for an electromagnetic field within a medium of permittivity ε and permeability μ is of the form

$$\boldsymbol{F} = \boldsymbol{F}_1\,(x - vt) + \boldsymbol{F}_2\,(x + vt).$$

Discuss the physical nature of this solution, stating the relation between the parameters v, ε and μ.
[7 marks]

1.8) The Cartesian components of the electric field strength $\boldsymbol{E}$ of a particular electromagnetic field are given as

$$E_x = 0; \quad E_y = E_{y0}\, e^{i(\omega t - kx)}; \quad E_z = E_{z0}\, e^{i(\omega t - kx)}\, e^{i\delta}.$$

Describe the various *polarisation states* of this wave field. Your answer should indicate the influence of the wave amplitudes E_{y0} and E_{z0} and the phase angle δ on the polarisation state.
[7 marks]

SECTION B — Answer TWO questions

2) Explain the meaning of the term *polarisation charge* as used in electrostatics.
[2 marks]

When a dielectric is placed in an electric field of strength $\boldsymbol{E}$, polarisation charges appear on its surface and throughout its volume. Prove that
a) the surface density of polarisation charge is $\sigma_P = \boldsymbol{P} \cdot \boldsymbol{n}$, where $\boldsymbol{n}$ is the unit normal to the surface.
b) the volume density of polarisation charge is $\rho_P = -\nabla \cdot \boldsymbol{P}$.
[10 marks]

A capacitor is constructed using two concentric metal cylinders, the inner cylinder having a radius of 1 cm and the outer cylinder a radius of 3 cm. The capacitor is completed by threading a dielectric tube with an inner radius of 1 cm and an outer radius of 2 cm onto the central electrode. The relative permittivity of this dielectric is $\varepsilon_r = 2.0$.
Prove that the capacitance per meter of length of this capacitor is

$$C = \frac{2\pi\varepsilon_0}{\log_e(3/\sqrt{2})}.$$

[10 marks]

If the charge on the inner cylinder is 1 nC per meter of length, and the outer electrode is earthed,
a) calculate the polarisation charge densities on the inner and outer surfaces of the dielectric.
b) show that the potential difference between the capacitor plates is 13.5 V.
[8 marks]

3) Write a brief account of the phenomenon of *ferromagnetism*.

[12 marks]

Discuss the meaning of the term *Ampèrian current*. Your answer should include a statement of the relations between the magnetisation $\boldsymbol{M}$ of the medium and the surface and volume Ampèrian current densities $\boldsymbol{J}_A$ and $\boldsymbol{\jmath}_A$. A mathematical derivation of these results is *not* required.

[8 marks]

A permanently magnetised sphere of radius a is located in free space. The magnetisation $\boldsymbol{M}$ of the sphere is uniform and points in the z-direction. Using spherical polar coordinates (r, ϑ, φ), where ϑ is measured from the positive z-direction, obtain formulas for the Ampèrian surface and volume current distributions.
Draw a sketch to illustrate the surface distribution.

[6 marks]

If $a = 5\,\text{cm}$ and $M = 10^6\,\text{A}\,\text{m}^{-1}$, prove that the Ampèrian current I_A flowing on the surface of the sphere is equal to 10^5 A.

[4 marks]

4) Explain the reason for the introduction of the *displacement current* in electromagnetic theory.

[6 marks]

The electric vector of an electromagnetic wave field propagating in an ideal dielectric medium of permittivity ε and permeability μ is given as

$$E_x = 0; \quad E_y = 0; \quad E_z = E_0 e^{i(\omega t - kx)}.$$

Use the wave equation to prove that $k^2 = \omega^2 \mu \varepsilon$.

[4 marks]

State the relations between the spatial frequency k and the propagation velocity v, the wavelength λ in the medium and the refractive index n of the medium.

[3 marks]

Show that the components of the magnetic field strength $\boldsymbol{H}$ of the above wave field are

$$H_x = 0; \quad H_y = -\sqrt{\frac{\varepsilon}{\mu}}\,E_z; \quad H_z = 0.$$

[6 marks]

Derive an equation for the *time-averaged* Poynting vector for this wave field, expressing the final result in terms of the r.m.s. value of E.

[6 marks]

If the r.m.s. value of E is $1\,\text{V}\,\text{m}^{-1}$, and the wave is propagating *in free space*, calculate the average value of the Poynting vector and the r.m.s. values of the magnetic field strength $\boldsymbol{H}$, the electric displacement $\boldsymbol{D}$ and the magnetic induction $\boldsymbol{B}$.

[5 marks]

5) Starting with Maxwell's equations, show that any electromagnetic field quantity $\boldsymbol{F}$ is a solution of the equation

$$\nabla^2 \boldsymbol{F} = \mu\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \mu\varepsilon \frac{\partial^2 \boldsymbol{F}}{\partial t^2}.$$

It is given that $\nabla \times \nabla \times \boldsymbol{F} = \nabla\nabla\cdot\boldsymbol{F} - \nabla^2\boldsymbol{F}$.

[6 marks]

Define the *metallic* and *dielectric* approximations that are used in electromagnetic theory.

[4 marks]

The relative permittivity of sea water is $\varepsilon_r = 80$ and its conductivity is $\sigma = 5\,\mathrm{S\,m^{-1}}$. The relative permeability is unity. For what range of frequencies does the metallic approximation apply?

[4 marks]

Consider a linearly polarised electromagnetic wave of frequency 10 kHz propagating vertically downwards in sea water. Show
a) a suitable solution for the electric field strength is

$$E_x = 0; \quad E_y = E_0 e^{i(\omega t - kx)}; \quad E_z = 0.$$

where x is the vertical distance measured from the sea surface.
b) the value of k is

$$k = \sqrt{-i\omega\mu\sigma} = (1-i)\sqrt{\frac{\omega\mu\sigma}{2}}.$$

[10 marks]

A radio receiver in a submarine can just detect an electric field strength of $10^{-9}\,\mathrm{V\,m^{-1}}$. The horizontal electric field strength at the sea surface above the submarine produced by a distant 10 kHz transmitter is $10^{-6}\,\mathrm{V\,m^{-1}}$. Calculate the maximum depth to which the submarine can submerge and still detect the radio signal.

[6 marks]