King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/2380 ELECTROMAGNETISM

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ε_0 =	8.854×10^{-12} Fm^{-1}
Permeability of free space	μ_0 =	$4\pi \times 10^{-7}$ Hm^{-1}
Speed of light in free space	c =	2.998×10^{8} ms^{-1}

Section A — Answer SIX parts of this section

1.1) Two metal plates of area 10 cm^2 are placed 5 mm apart. A dielectric, with ε_r = 2.5 and thickness 3 mm, is placed mid-way between the plates. A potential of 100 V is applied across the plates. Show that the capacitance of the arrangement is 2.77 pF and the surface charge density on the metal plates is 277 nCm^{-2}. Ignore edge effects.

[7 marks]

1.2) Show that in a conducting medium (conductivity σ and permitivity ε) any free charge deposited leaks away with a characteristic time $\tau = \varepsilon/\sigma$, known as the relaxation time. For an isolated charged conductor where will the charge eventually reside at $t >> \tau$?

[7 marks]

1.3) Give a brief account of the phenomenon of diamagnetism.

[7 marks]

1.4) State how the vector **B** can be expressed in terms of a magnetic vector potential **A**. Also state how **E** can be expressed in terms of **A** and a scalar potential V. Explain why it is useful to introduce **A** and V.

[7 marks]

1.5) Explain what is meant by the *skin effect*. The skin depth δ is determined by $\delta = \sqrt{2/\mu_r \mu_0 \sigma\omega}$ where σ is the conductivity and ω the angular frequency. Calculate the skin depth for aluminium at 50 Hz. (The resistivity of aluminium = 1.5×10^{-8} Ωm and μ_r = 1).

[7 marks]

1.6) In the radiation zone the magnetic field produced by a z-directed current dipole located at (0,0,0) in free space is (in spherical polar coordinates)

$$H_\phi = \frac{ikA\exp[i(\omega t - kr)]\sin\theta}{r} \quad , \quad H_\theta = 0 \quad , \quad H_r = 0$$

where A is a constant and all terms have their usual meaning. Use Maxwell's $\nabla\times\mathbf{H}$ equation to show that

$$E_\theta = \frac{ik^2 A\exp[i(\omega t - kr)]\sin\theta}{\omega\varepsilon_0 r} \qquad \text{and} \qquad E_r = -\frac{2kA\exp[i(\omega t - kr)]\cos\theta}{\omega\varepsilon_0 r^2} \quad .$$

Comment on the form of both components.

[Note since $H_r = H_\theta = 0$ then $\nabla\times\mathbf{H} = \hat{r}\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(H_\phi \sin\theta\right)\right] + \hat{\theta}\left[-\frac{1}{r}\frac{\partial}{\partial r}\left(rH_\phi\right)\right]$]

[7 marks]

1.7) State Ampère's Law. Explain the reason for the introduction of the *displacement current* in electromagnetic theory.

[7 marks]

1.8) Derive the boundary conditions for the electrostatic **E** and **D** fields at the interface between two dielectric media.

[7 marks]

Section B — Answer TWO questions from this section

2.(a) What is the condition that must be true for the electric field vector **E** to be written as the gradient of a scalar function? Show how writing **E** in this form leads to Poisson's equation for the electric potential.

[6 marks]

(b) An uncharged conducting cylinder of infinite length and radius a is placed in a previously uniform electric field (magnitude E_0) with its axis at right angles to the electric vector. Show that

$$V = Ar\cos\theta + (B/r)\cos\theta$$

is a possible distribution of the electric potential in cylindrical polar co-ordinates (r,θ,z). Obtain expressions for the constants A and B in terms of E_0 and a.

[12 marks]

(c) Give a brief explanation of the physical significance of the two terms in the expression for V.

[4 marks]

(d) Derive an expression for the distribution of charges induced on the conductor. Sketch this distribution together with the lines of electric field around the cylinder.

[8 marks]

[In cylindrical polar co-ordinates $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$]

3.(a) Show that the quantity ½**E•D** is an expression for the energy density (U) of an electrostatic field, where **E** is the electric field vector and **D** the electric displacement.

[6 marks]

(b) Show that a change in U is given by $\delta U = \mathbf{E} \bullet \delta\mathbf{D}$.

[4 marks]

(c) Justify the use of an energy flow vector for electromagnetic fields given by the Poynting vector **E**×**H**.

[12 marks]

(d) A parallel plate capacitor has circular plates of radius a separated by an air space d ($d \ll a$ so that edge effects can be ignored). During a certain time interval, the capacitor is charged by a constant current I flowing in the circuit to which the capacitor is connected. Obtain expressions for the rate of increase of energy stored in the capacitor by integration of the Poynting vector over a suitable surface.

[8 marks]

4. Use Maxwell's equations to obtain the wave equation for electromagnetic waves in a charge free, linear and isotropic medium.

(You may use the vector identity $\nabla \times \nabla \times \mathbf{F} = \nabla\nabla \bullet \mathbf{F} - \nabla^2 \mathbf{F}$.)

[6 marks]

The magnetic vector of an electromagnetic wave in free space is given as

$$H_x = H_z = 0, \quad H_y = H_0 \cos(\omega t - kx)$$

(a) Use the wave equation to prove that $k^2 = \omega^2 \mu_0 \varepsilon_0$. What is the significance of the parameter k?

[4 marks]

(b) Derive a formula for the electric field strength **E**.

[5 marks]

(c) Derive an expression for the wave impedance. Calculate the magnitude of it in free space.

[5 marks]

(d) Briefly discuss the physical nature of this wave.

[4 marks]

(e) Derive an equation for the time averaged power density in this wave field.

[6 marks]

5.(a) How are the magnetic field strength **H**, the magnetic flux density **B** and the magnetisation **M** related? What is the magnetic susceptibility and how is it defined?

[4 marks]

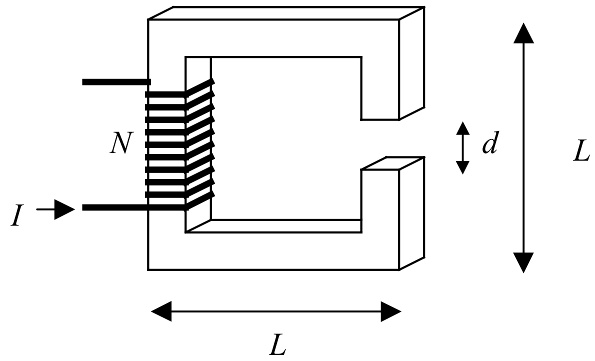
(b) Describe what is meant by the term *hysteresis* in relation to a *ferromagnetic* material. What characteristics should a *ferromagnetic* material have if it is to be used to make a permanent magnet?

[6 marks]

(c) A toroid having a soft iron core of square cross section and relative permeability μ_r is wound with N closely spaced turns of wire carrying a current I. Using Ampère's Law and the result of part (a) derive an expression for the magnetisation **M** inside the iron.

[8 marks]

(d) An electromagnet is made by wrapping a current carrying coil *N* times around a C-shaped piece of iron (relative permeability $\mu_r >> 1$) as shown in the figure below. The current in the coil is *I*, the width of the gap is *d*, and the length of the side of the "C" is *L*. It may be assumed that $L >> d$. Derive an expression for the B-field in the gap explaining any assumptions you make.



[12 marks]