King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/2380 ELECTROMAGNETISM

Summer 2000

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ε_0	$= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) Give a definition of the electric field strength $\mathbf{E}$.
Electric charge is distributed through a free-space volume v with a volume charge density ρ. Use the Coulomb law of force between two electrical charges to derive a formula for the electric field strength produced by this volume distribution.
[7 marks]

1.2) Discuss the characteristics of a *dielectric*.
Give a brief account of the phenomenon of *dielectric breakdown*.
[7 marks]

1.3) Derive an expression for the external work done in charging a capacitor of capacitance C by a source which produces a potential difference V.
[7 marks]

1.4) Give an expression for the magnetic moment $\mathbf{m}$ of a small electrical circuit carrying a current I.
Describe briefly how this result may be applied to calculate the magnetic effects of an electrical circuit of any size.
[7 marks]

1.5) Give a brief account of the phenomenon of *paramagnetism*.
[7 marks]

1.6) State Maxwell's equations for an electromagnetic field propagating in a linear medium of relative permittivity ε_r, relative permeability μ_r and electrical conductivity σ, defining all of the symbols used.
[7 marks]

1.7) Explain what is meant by the term *plane electromagnetic wave*.
A plane electromagnetic wave in an isotropic medium is said to propagate in the *transverse* electromagnetic mode. What does the word *transverse* imply in this context?
[7 marks]

1.8) Define the terms *retarded time* and *radiation* (or *far-field*) *zone* as used in the theory of antennas.
[7 marks]

SECTION B – Answer TWO questions

2) Define the term *polarisation* $\mathbf{P}$ as used in the study of dielectrics.

[2 marks]

State the relation between the volume polarisation charge density ρ_P and $\mathbf{P}$.

[2 marks]

Starting with the expression $\varepsilon_0 \nabla \cdot \mathbf{E} = \rho$, derive the relationship $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ between the electrical displacement $\mathbf{D}$, the electric field strength $\mathbf{E}$ and $\mathbf{P}$. State the significance of $\mathbf{D}$ in the theory of the electrostatics of material media.

[6 marks]

A point charge Q is placed in a homogeneous dielectric whose relative permittivity is ε_r. By the application of Gauss's law or otherwise, obtain expressions for $\mathbf{D}$ and $\mathbf{E}$ at a distance r from Q and show that,

$$\mathbf{P} = \frac{(\varepsilon_r - 1)}{\varepsilon_r} \frac{Q}{4\pi r^2} .$$

[6 marks]

Repeat the above calculation for a situation in which the dielectric is *inhomogeneous* with a relative permittivity which varies only with r according to $\varepsilon_r = (1 + a/r)$, where a is a constant.

[8 marks]

Obtain formulas for the volume density of polarisation charge ρ_P at a distance r from Q for both the homogeneous and inhomogeneous dielectrics of the previous two paragraphs. It is given that, in spherical polar co-ordinates,

$$\nabla \cdot \mathbf{P} = \frac{1}{r^2} \frac{\partial \left(r^2 P_r\right)}{\partial r} .$$

[6 marks]

3) State Ampères circuital law.

[2 marks]

Define the terms *Ampèrian current*, *surface density of an Ampèrian current* ($\mathbf{J}_A$) and *volume density of an Ampèrian current* ($\mathbf{j}_A$).

[6 marks]

Prove that, when a body acquires a magnetisation $\mathbf{M}$ by virtue of an applied magnetic field of strength $\mathbf{H}$, the magnetisation may be represented by a distribution of Ampèrian currents at each point on its surface with a surface density given by $\mathbf{J}_A = \mathbf{M} \times \mathbf{n}$. Here $\mathbf{n}$ is the outwardly directed unit normal to the surface at the point.

[10 marks]

The axis of a hollow, straight, iron cylindrical pipe defines the z-direction of a cylindrical co-ordinate system (r, ϕ, z). The inner radius of the pipe is a and the outer radius is b. The relative permeability of the iron is μ_r. A steady electrical current I flows in the z-direction in the metal of the pipe.

For this situation:

a) Use the Ampère circuital law to obtain a formula for the magnetic field strength $\mathbf{H}$ in the metal of the pipe at a distance r from the axis of symmetry.

[3 marks]

b) Hence prove that the magnetisation $\mathbf{M}$ of the iron is given by

$$\mathbf{M} = \hat{\phi}\,(\mu_r - 1)\,\frac{I}{2\pi r}\,\frac{r^2 - a^2}{b^2 - a^2} \qquad a \leq r \leq b\,,$$

where $\hat{\phi}$ is a unit vector in the ϕ-direction.

[4 marks]

c) Derive a formula for the surface Ampèrian current density. Show that this is always zero on the inner surface of the pipe. If I is 1 A, $\mu_r = 1001$ and $b = 10$ mm, show that on the outer surface of the pipe $\mathbf{J}_A$ is in the $-z$ direction and has a magnitude of 15,915 A/m.

[5 marks]

4) Explain carefully the meaning of the term *state of polarisation* as applied to an electromagnetic wave. Illustrate your answer by drawing a sketch showing *one* possible state of polarisation, stating the conditions under which your choice is produced.

[6 marks]

Define the *Poynting vector*.
What precaution must you take in evaluating the time-averaged Poynting vector for an electromagnetic field which is expressed in the complex (or cissoidal) form?

[4 marks]

The Cartesian components of the magnetic vector of an electromagnetic field *in free space* are given as

$$H_x = 0; \quad H_y = H_0\, e^{i(\omega t - kx)}; \quad H_z = H_0\, e^{i(\omega t - kx - \pi/2)}\,,$$

where H_0, ω and k are constants.

a) What is the *plane of polarisation* of this wave and in which direction is the wave propagating?

[2 marks]

b) What is the state of polarisation of this wave?

[3 marks]

c) Use Maxwell's curl **H** equation to calculate the electric field strength **E** of the wave and draw a sketch to illustrate the polarisation of the electric field.

[9 marks]

d) Prove that the Poynting vector of the given wave is a constant and discuss why this result is obtained.

[6 marks]

5) In electromagnetism, a magnetic vector potential $\mathbf{A}$ may be defined so that the magnetic induction $\mathbf{B}$ is given as $\mathbf{B} = \nabla \times \mathbf{A}$. Why is it possible to express $\mathbf{B}$ in this way?

Use Maxwell's curl $\mathbf{E}$ equation to prove that the electric field strength $\mathbf{E}$ can be expressed in the form

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}\,.$$

[6 marks]

In source-free regions the vector potential $\mathbf{A}$ satisfies the homogeneous Helmholtz equation

$$(\nabla^2 + k^2)\psi = 0\,,$$

where ψ is any Cartesian component of $\mathbf{A}$ and $k^2 = \omega^2\mu\varepsilon$.

Consider a Hertzian dipole of current moment $I\,\delta z$ at the origin of a Cartesian co-ordinate system (x, y, z). For this source it is given that $\mathbf{A}$ only has a z-component A_z. By finding a solution to the Helmholtz equation for A_z, show that, in spherical polar co-ordinates, the ϕ-component of $\mathbf{B}$ is given by

$$B_\phi = a\left(ik + \frac{1}{r}\right)\frac{e^{-ikr}}{r}\sin\theta\,,$$

where a is a constant.

The following two formulas may be assumed:

$$\nabla^2 A_z = \frac{1}{r}\frac{\partial^2 (rA_z)}{\partial r^2}$$

and

$$(\nabla \times \mathbf{A})_\phi = \frac{1}{r}\left(\frac{\partial (rA_\theta)}{r} - \frac{\partial A_r}{\partial \theta}\right)\,.$$

[15 marks]

Give a *qualitative* description of the characteristics of the electromagnetic fields of this source *in the radiation zone*. Your answer should include consideration of the wave impedance of the fields and a statement of the direction of maximum power radiation. Mathematical analyses are *not* required.

[9 marks]