

King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP1600 Physical Basis of Astronomy

Summer 2006

Time allowed: THREE Hours

**Candidates should answer ALL parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.**

The approximate mark for each part of a question is indicated in square brackets.

**You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.**

**TURN OVER WHEN INSTRUCTED
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Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	F m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Speed of light in free space	c	=	2.998×10^8	m s^{-1}
Gravitational constant	G	=	6.673×10^{-11}	$\text{N m}^2 \text{kg}^{-2}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\text{W m}^{-2} \text{K}^{-4}$
Gas constant	R	=	8.314	$\text{J mol}^{-1} \text{K}^{-1}$
Avogadro constant	N_A	=	6.022×10^{23}	mol^{-1}
Molar volume of ideal gas at STP		=	2.241×10^{-2}	m^3
One standard atmosphere	P_0	=	1.013×10^5	N m^{-2}
Radius of Sun	R_\odot	=	6.961×10^8	m
Mass of Sun	M_\odot	=	1.989×10^{30}	kg
Surface Temperature of Sun	T_\odot	=	5800	K
Wavelength of maximum flux (Sun)	$\lambda_{\text{MAX}}(\odot)$	=	5500	\AA
Solar bolometric magnitude	$M_{\text{bol},\odot}$	=	+4.75	
V filter zero point	$F_\lambda(V=0)$	=	3.64×10^{-12}	$\text{W m}^{-2} \text{\AA}^{-1}$
V filter central wavelength	$\lambda_0(V)$	=	5500	\AA
V filter bandwidth	$\Delta\lambda(V)$	=	890	\AA

SECTION A – Answer ALL parts of this section

- 1.1) Define the sidereal and synodic period of a planet. Derive the relation between the sidereal period (P) of a superior planet as a function of the synodic period (S) and the period (E) of the Earth's orbit.

[6 marks]

- 1.2) Define twilight and list the three standard choices of twilight. Why does twilight last longer at higher latitudes? How long does twilight last on the Moon?

[6 marks]

- 1.3) Consider a star with an apparent magnitude $m_V = 11$.

Show that we receive a flux of $400 \text{ photons s}^{-1}\text{m}^{-2} \text{ \AA}^{-1}$ from this star. A telescope with a 50 cm aperture has a total throughput $T_V = 0.3$ in the V band. It uses a CCD detector with quantum efficiency of 80% and you may assume that the contribution to the noise from the sky, read noise and dark current are negligible. Calculate the signal to noise ratio after an exposure of 2 seconds.

[10 marks]

- 1.4) Define hydrostatic equilibrium. Write down the equation of hydrostatic equilibrium.

Consider a star with a constant density ρ_0 . Find the central pressure (p at $r = 0$) as a function of the stellar mass M and radius R .

[10 marks]

- 1.5) For the three observatories at the latitudes listed below, draw a celestial sphere with the zenith, celestial pole, celestial equator and horizon. For each one show a star with hour angle $H = 0$ hours and declination $\delta = +45^\circ$.

a) Latitude $\phi = 45^\circ\text{N}$

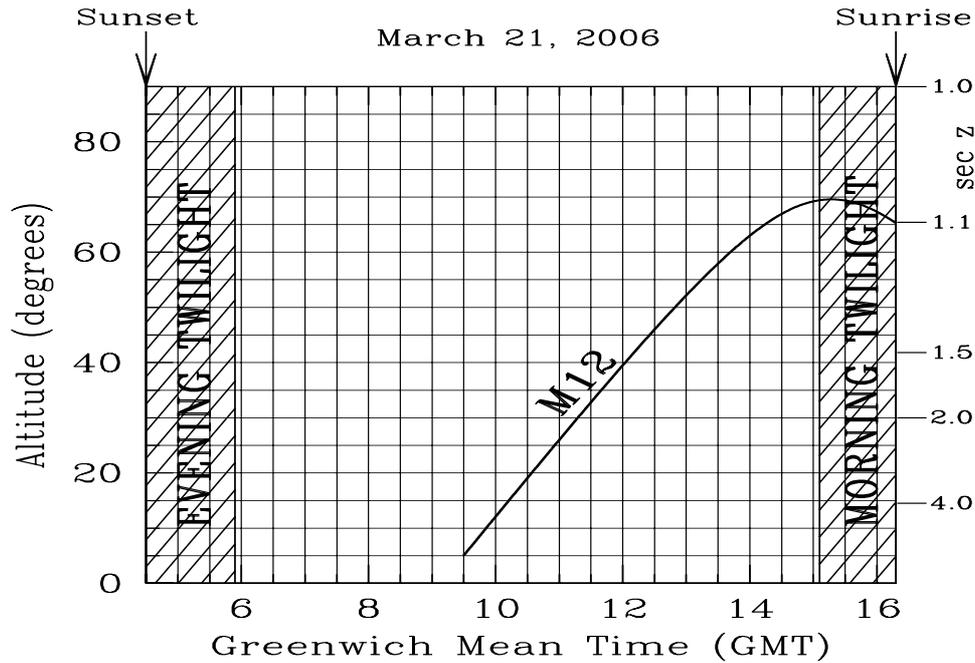
b) Latitude $\phi = 45^\circ\text{S}$

c) Latitude $\phi = 80^\circ\text{N}$

[8 marks]

SECTION B – Answer TWO questions

- 2) The figure below shows the altitude of M12 (which is a globular cluster) above the horizon throughout the night of the 21st of March at some observatory in the Northern hemisphere. The equatorial coordinates of M12 are RA=16h47m and Dec=-01°57'.



- a) Determine the latitude and longitude of the observatory, assuming the equation of time is zero. [6 marks]
- b) What is the local sidereal time at midnight? [4 marks]
- c) The figure shows that March 21st is not the best of days to observe the target. Estimate the ideal **date** to observe M12. [5 marks]
- d) A globular cluster is an old stellar population: a system of stars all formed at the same time, about 10^{10} years ago. Why are there no massive stars in M12? Show in a Hertzsprung-Russell diagram the following stellar phases: low-mass main sequence, red giant branch, horizontal branch, and white dwarf. Briefly explain the stellar structure of these phases. [10 marks]
- e) Explain why the secant of the zenith angle ($\sec z$) is important when computing atmospheric extinction. [5 marks]

3) Consider a spaceship on a circular orbit with radius 1 AU around a star with mass $2M_{\odot}$.

a) Find the period of the circular orbit described above.

[5 marks]

b) The effective potential is $\phi(r) = \lambda^2/2r^2 - GM/r$. What is meant by effective potential? Describe the meaning of both terms in $\phi(r)$. Sketch the effective potential as a function of radius and identify the circular, elliptical, parabolic and hyperbolic orbits.

[5 marks]

c) Show that, for any elliptical orbit, the radial component of the velocity dr/dt is zero only at its aphelion and perihelion.

[5 marks]

d) A rocket is fired so that the tangential velocity of the spaceship is increased by 10%. Determine the new orbit by calculating its new semi-major axis and eccentricity. Explain **qualitatively** what would happen if the rocket *decreases* the tangential velocity of the spaceship by 10%.

[10 marks]

e) Consider the initial circular orbit. There is a sudden mass loss process in the star so that its mass instantaneously drops to some value M' and the spaceship's orbit becomes parabolic (only because of this mass loss). Determine M' .

[5 marks]

P.S. The orbit of a body in a gravitational field can be written in polar coordinates as: $r = r_0/(1 + \epsilon \cos \theta)$, where ϵ is the eccentricity, which relates the semi-major (a) and the semi-minor (b) axes of the orbit by $b^2 = a^2(1 - \epsilon^2)$. The semi-latus rectum r_0 defines the size of the orbit, and can be written in terms of the the angular momentum per unit mass of the orbiting body – $\lambda = v_{\perp}r$ – and the mass of the star: $r_0 = \lambda^2/GM$. It is also related to the semi-major axis by: $r_0 = a(1 - \epsilon^2)$.

- 4.a) Define absolute magnitude. Show that a star with absolute magnitude in the V band M_V , at a distance d (in parsec) has an apparent magnitude given by:

$$m_V = M_V + 5 \log(d) - 5.$$

[5 marks]

- b) Define Wien's law. Calculate the surface temperature of an A-type star by comparing its wavelength of maximum emission $\lambda_{\text{MAX}}(\text{A}) = 3200 \text{ \AA}$, with that of the Sun.

[5 marks]

- c) An A-type star with apparent magnitude $m_V = 15.60$ is located 100 pc away. Calculate its absolute magnitude.

The bolometric correction for this type of star is -0.2 . Calculate its radius as a fraction of the solar radius R_\odot , and identify the type of star.

[10 marks]

- d) Consider the following two A-type stars: Vega ($m_V = 0.0$, annual parallax $0.123''$) and Deneb ($m_V = +1.25$, distance 490 pc). Compute the ratio of the **observed** fluxes. Compute the ratio of luminosities and radii, assuming they have the same surface temperature and the same bolometric correction.

Determine the radius of Vega in units of R_\odot , the radius of the Sun. Explain briefly whether Vega and Deneb are main sequence or giant stars.

[10 marks]