

# King's College London

UNIVERSITY OF LONDON

**This paper is part of an examination of the University counting towards the award of a degree. Examinations of the University are governed by the Senate Regulations.**

**B.Sc. EXAMINATION**

**CP/1490 THE STRUCTURE OF MATTER**

**JANUARY 1999**

**Time allowed: THREE HOURS**

**Candidates must answer any SIX parts of SECTION A,  
and TWO questions from SECTION B.**

**The approximate mark for each part of a question is indicated in the square brackets.**

**Separate answer books must be used for each section of the paper.**

**TURN OVER WHEN INSTRUCTED**

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## Constants

Electron charge (e):	$1.602 \times 10^{-19} \text{ C}$
Electron mass ( $m_e$ ):	$9.109 \times 10^{-31} \text{ kg}$
Planck's constant (h):	$6.626 \times 10^{-34} \text{ J s}$
Speed of light in a vacuum (c):	$2.998 \times 10^8 \text{ m s}^{-1}$
Atomic mass unit (amu):	$1.662 \times 10^{-27} \text{ kg}$ or $931.5 \text{ MeV}/c^2$
Boltzmann's constant (k):	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Electric permittivity of free space ( $\epsilon_0$ ):	$8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Acceleration due to gravity at sea level (g):	$\sim 9.82 \text{ m s}^{-2}$
Natural log of 2 (ln(2)):	$\sim 0.6931$

## SECTION A - Answer any SIX questions from this section

- 1.1) Sketch Milliken's apparatus for measuring the charge on an electron using negatively charged oil drops. Indicate the direction in which the oil drops move with the electric field switched on and indicate the forces which are acting on the drops. How could the electron charge be confirmed by another experiment? [7 marks]
- 1.2) CO<sub>2</sub> molecules have four modes of vibration. Explain why the infra-red absorption spectrum shows only two peaks. [7 marks]
- 1.3) What is meant by *tunnelling current*? Draw a labelled diagram showing how the tip is kept at a constant distance from the surface of the sample in a scanning tunnelling microscope. [7 marks]
- 1.4) Draw an energy level diagram which indicates the transitions which give rise to the L<sub>α</sub>, L<sub>β</sub>, K<sub>α</sub> and K<sub>β</sub> peaks in the emission spectrum of a copper target x-ray source. What limits the brightness of electron bombardment x-ray sources? [7 marks]
- 1.5) An electron travels in a circular path of radius 0.03 m in a plane normal to a magnetic field of 0.005 Tesla. Calculate the de Broglie wavelength of the electron. [7 marks]
- 1.6) After the "big bang", the first atoms to form were hydrogen and helium. What average temperature did the universe have to cool down to in order for atomic hydrogen to form? [7 marks]
- 1.7) The potential energy between two atoms, separated by a distance  $r$ , can be described by

$$V(r) = \epsilon \left[ \left( \frac{a_0}{r} \right)^{12} - 2 \left( \frac{a_0}{r} \right)^6 \right],$$

where  $a_0$  is the equilibrium separation distance at low temperature, and  $\epsilon$  is the potential energy at this separation. Show that when the separation of the molecules is equal to  $a_0$ , the force acting is zero. Show which of the terms in the equation contributes most to the binding energy at the equilibrium separation. [7 marks]

- 1.8) A rock sample contains 5 atomic parts per million of <sup>238</sup>U and 2 atomic parts per million of <sup>206</sup>Pb. <sup>206</sup>Pb is the stable isotope at the end of the <sup>238</sup>U decay series. The decay series from <sup>238</sup>U to <sup>206</sup>Pb is dominated by the half life of <sup>238</sup>U. The half-life of <sup>238</sup>U is 4500 × 10<sup>6</sup> y. Determine the age of the rock. [7 marks]

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## SECTION B - Answer any TWO questions

- 2) Alpha particles of energy of 2 MeV are incident on a thin gold foil (atomic number 79). Considering an alpha particle which is scattered through 180 degrees, calculate:

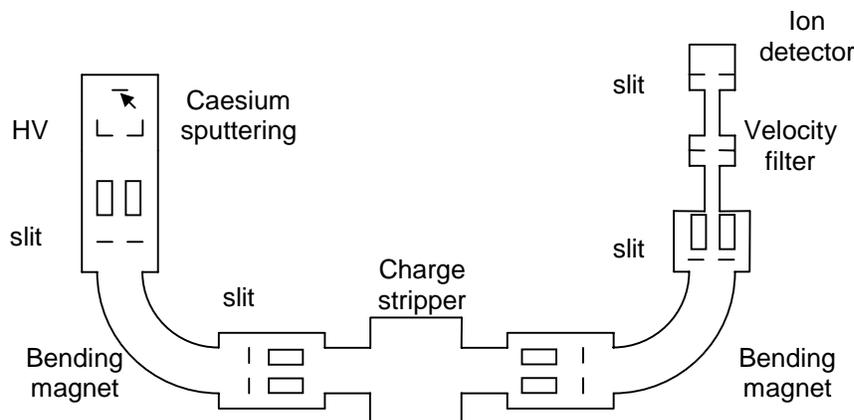
- the initial velocity of the alpha particle; [5 marks]
- the distance of closest approach of the alpha particle with the gold nucleus; [7 marks]
- the velocities of the gold nucleus and the alpha particle after the scattering event. [11 marks]

Thallium ( $^{201}_{81}\text{Tl}$ ) can be produced from alpha particle collisions with gold ( $^{197}_{79}\text{Au}$ ) nuclei. Calculate the minimum energy an alpha particle must have for this to happen.

[7 marks]

Alpha particle mass,  $m_{\alpha} = 6.6465 \times 10^{-27}$  kg, gold nucleus mass,  $m_{\text{Au}} = 3.27089 \times 10^{-25}$  kg, thallium nucleus mass  $m_{\text{Tl}} = 3.33739 \times 10^{-25}$  kg.

- 3) Fragments of once-living organisms may be dated by using an accelerator mass spectrometer (AMS). The essential components of an AMS are shown below. Briefly describe how this instrument works and indicate how it may be used to measure the relative amounts of  $^{14}\text{C}$ ,  $^{12}\text{C}$  and  $^{13}\text{C}$  in a sample and therefore determine the age of the sample.



[10 marks]

A carbon sample is extracted from a bone found in a cave. The total mass of carbon in the sample is 0.05g. The  $\beta^-$  decay of the sample was measured in a scintillation based analyser over 24 hours during which there were 512 recorded counts. Given that  $^{14}\text{C}$  and  $^{12}\text{C}$  are present in living matter in the atomic ratio  $10^{-12}$  to one and the half life of  $^{14}\text{C}$  is 5730 years, calculate the age of the bone sample from the decay analysis.

[10 marks]

A second sample (mass 0.05g) from the same bone was analysed by AMS. The ratio of  $^{14}\text{C}:^{12}\text{C}$  was found to be  $5.87 \times 10^{-13}:1$ . Calculate the age of the bone from this result. Give possible reasons why the two techniques give different answers.

[10 marks]

**SEE NEXT PAGE**

- 4) A pendulum in a museum consists of a 2 kg mass suspended from a 20m long metal wire which has a Young's modulus of  $1.30 \times 10^{11} \text{ Nm}^{-2}$ . The wire suffers a strain of 0.0192% when the mass is attached. Calculate the diameter of the wire and its extension under the load. [5 marks]

Which would have the greater effect on the period of swing on the pendulum, a temperature change of  $5^\circ \text{C}$  or increasing the mass of the weight to 4 kg? You may assume that the period of the pendulum ( $\tau$ ) can be calculated from  $\tau = 2\pi\sqrt{l/g}$ , where  $l$  is the length of the wire and  $g$  is the acceleration due to gravity. The metal's coefficient of expansion is  $3 \times 10^{-4} \text{ m}^{-1} \text{ }^\circ\text{C}^{-1}$ . [7 marks]

A sample of the wire is studied by diffraction using x-rays which of wavelength  $4 \times 10^{-10} \text{ m}$ . The largest Bragg angle observed is  $53.13^\circ$ . Calculate the distance in the loaded direction by which each bond is stretched when the 20m wire is loaded by the 2kg weight. [10 marks]

Calculate the minimum x-ray energy which can be used to measure the atomic spacing. [8 marks]

- 5) In the Bohr model of the hydrogen atom, the electron orbits have quantised angular momentum equal to  $n\hbar$ , where  $n$  is an integer and  $\hbar$  is Planck's constant divided by  $2\pi$ . Show that the radius ( $r_n$ ) of the  $n$ th orbit in a hydrogen atom is given by

$$r_n = \frac{4\pi \epsilon_0 \hbar^2 n^2}{me^2}$$

[6 marks]

By considering the kinetic and potential energies that an electron has in the hydrogen atom, show that the energy of the  $n$ th state is

$$\xi_n = -\frac{e^4 m}{2(4\pi\epsilon_0)^2 \hbar^2 n^2}$$

[9 marks]

Hydrogen gas is contained in a tube equipped with electrodes which can be used to apply an electric field to the gas. The mean free path of ionised hydrogen in the tube is  $1 \times 10^{-4} \text{ m}$ . Calculate the value of the electric field where collisions between ionised and atomic hydrogen will result in hydrogen emission arising from transition between the  $n=2$  and  $n=1$  orbitals.

[9 marks]

Calculate the value of the electric field which would lead to an avalanche in the tube.

[6 marks]

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