King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP1490 Structure of Matter

Summer 2006

Time allowed: THREE Hours

Candidates should answer ALL parts of SECTION A, and no more than TWO questions from SECTION B. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ε_0	=	8.854×10^{-12}	$\mathrm{F\ m^{-1}}$
Permeability of free space	μ_0	=	$4\,\pi \times 10^{-7}$	$\mathrm{H\ m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\ s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\ K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\ m^{-2}\ K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\ m^{-2}}$
Acceleration due to gravity	g	=	9.810	$\mathrm{m\ s^{-2}}$

SECTION A – Answer ALL parts of this section

1.1) Describe briefly Rutherford's alpha-particle scattering experiment. Explain why the results of this experiment led Rutherford to propose the nuclear model for the atom.

[6 marks]

1.2) Determine the relation between the kinetic energy and the de Broglie wavelength of a particle moving with a velocity much less than the speed of light. Calculate the de Broglie wavelength of an electron which has been accelerated through a potential difference of 100 V.

[7 marks]

1.3) Identify the six types of lepton and describe their properties.

[5 marks]

1.4) Explain why, in alpha decay, short half-lives correspond to large disintegration energies. Determine the energy released during the alpha decay of ^{238}U to ^{234}Th.
The mass of ^{238}U is 238.0508m_u, the mass of ^{234}Th is 234.0436m_u and the mass of ^{4}He is 4.0026m_u.

[8 marks]

1.5) A plastic rod of diameter 2 mm is stretched along its length by a force of 100 N. Determine the change in diameter of the rod.
The Young modulus for the plastic rod is 10^9 Nm^{-2} and the Poisson ratio is 0.3.

[7 marks]

1.6) The mean free path length λ of molecules of diameter d in an ideal gas is given by the expression

$$\lambda = \frac{1}{\pi n d^2} \ ,$$

where n is the number of molecules per unit volume. Nitrogen gas is held in a container at a temperature of 100 K and a pressure of 10^4 Pa. Calculate the mean free path length and root-mean-squared velocity of the nitrogen molecules.
Take the mass and diameter of a nitrogen molecule to be 28m_u and 3×10^{-10} m, respectively.

[7 marks]

SECTION B – Answer TWO questions

2 a) State Bohr's postulates for the quantum theory of the atom. State the quantisation law for angular momentum in the Bohr model of the atom, in terms of the principal quantum number n.

[6 marks]

b) Show that the kinetic energy K of an electron moving in a circular orbit of radius r and angular momentum L is given by

$$K = \frac{L^2}{2m_e r^2} \ .$$

Show that the kinetic energy K and potential energy V of an electron in a circular orbit around a positive nucleus are related by the equation $V = -2K$.

Hence show that the quantised total energy levels E_n of the electron in the hydrogen atom are given by

$$E_n = -\frac{m_e e^4}{8\varepsilon_0{}^2 h^2 n^2} \ .$$

[15 marks]

c) Determine the energy difference between the n = 1 and n = 2 energy levels, and between the n = 2 and n = 3 energy levels of the hydrogen atom.

Measurements in the infra-red part of the emission spectrum of hydrogen show the presence of series of spectral emission lines. The three longest wavelength lines of a particular series are 1.875 μm, 1.282 μm and 1.094 μm. Determine the principal quantum numbers of the atomic energy levels involved in the emission of each of these three spectral lines.

[9 marks]

3 a) Describe the mechanisms which lead to the production of X-rays when a material is irradiated by high energy electrons, indicating how the X-ray emission relates to the energy of the electrons, and to the energy-level structure of the material. Sketch a typical graph of the intensity of the radiation as a function of X-ray photon energy.

[12 marks]

b) A beam of radiation of wavelength λ is incident at an angle θ to the planes of a crystal. Within each crystal plane the atoms are equally spaced and the crystal planes are separated by a distance d. Show that a strong reflection, with an angle θ between the reflected beam direction and the planes of the crystal, could be observed for angles θ given by

$$n\lambda = 2d\sin\theta \ ,$$

where the integer n is the diffraction order.

X-rays of wavelength 0.072 nm are directed towards a crystal and a strong first order reflection is found for an angle $\theta = 7.6°$. Calculate the separation of atomic planes in the crystal.

[8 marks]

c) A reactor produces neutrons with a distribution of energies such that the neutrons may be described as an ideal gas with a temperature of 300 K. A beam of neutrons is directed towards a calcium fluoride crystal with a 0.19 nm separation of atomic planes, to give a narrow range of neutron wavelengths for the Bragg diffracted beam. Determine the angle θ with which the crystal should be oriented to the neutron beam to achieve the most intense neutron beam. Determine the two closest wavelengths which may also be present in the diffracted neutron beam.

[10 marks]

4 a) The potential energy $V(r)$ of an ion in a crystal of Na^+Cl^- is given by

$$V(r) = \frac{\alpha e^2}{4\pi\varepsilon_0 a_0}\left[\frac{1}{10}\left(\frac{a_0}{r}\right)^{10} - \frac{a_0}{r}\right],$$

where r is the nearest-neighbour distance between ions in the crystal and a_0 is the equilibrium separation. Sketch the form of the potential V(r) and explain the physical origins of the first and second terms in the equation and of the Madelung constant α.

Show that the minimum potential energy occurs when $r = a_0$.

[12 marks]

b) In NaCl the Madelung constant α = 1.748 and the equilibrium separation of ions a_0 = 0.28 nm. Calculate the energy in eV required to completely remove an ion from a crystal of NaCl.

[4 marks]

c) The NaCl crystal is compressed isotropically, such that all the bonds in the crystal undergo the same compressive strain. If the separation between ions is reduced by a small amount x from the equilibrium separation a_0 ($x \ll a_0$), show that the spring constant k of the ionic bonds (the force per unit compression) is given by the expression

$$k = \frac{9\alpha e^2}{4\pi\varepsilon_0 a_0^{\;3}} \quad .$$

[7 marks]

d) A perfect NaCl crystal under tensile stress will fracture when the forces between the ions are no longer sufficient to balance the applied force. Determine the strain at which this fracture will occur.

[7 marks]