King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP1490 Structure of Matter

Summer 2004

Time allowed: THREE Hours

Candidates should answer no more than SIX parts of SECTION A, and no more than TWO questions from SECTION B. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_{e}	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_{u}	=	1.661×10^{-27}	kg
Proton rest mass	m_{p}	=	1.673×10^{-27}	kg
Neutron rest mass	m_{n}	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_{B}	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_{A}	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

SECTION A – Answer SIX parts of this section

1.1) Sketch a labelled diagram of Millikan's oil drop experiment and explain how it may be used to demonstrate that there is a fundamental unit of charge.

[7 marks]

1.2) High resolution measurements of atomic emission spectra reveal fine structure in which each spectral line is split into two or more components. Describe the origin of this fine structure.

[7 marks]

1.3) Calculate the wavelengths of electrons and photons having the same energy of 10 eV.

[7 marks]

1.4) A mass of 100 kg is suspended from a copper rod of unloaded length 1 m and diameter 1 cm. An identical mass of 100 kg is suspended from a second rod of the same unloaded length 1 m but a diameter of 5 mm. Determine the difference in extension of the two rods. [The Young modulus of copper is $1.3 \times 10^{11}\,\mathrm{N\,m^2}$]

[7 marks]

1.5) Derive the Bragg diffraction law for X-rays of wavelength λ scattered by a crystal with spacing d between the planes of atoms:

$$2d \sin\theta = n\lambda \quad .$$

X-rays of wavelength 0.30 nm are diffracted in first order through an angle of 65 degrees by a crystal of NaCl. Determine the spacing between the planes of atoms in the crystal.

[7 marks]

1.6) Describe the processes which give rise to the K_α, K_β and L_α peaks in the X-ray emission spectrum produced when energetic electrons are incident on a metal target. Indicate how the energy of the K_α peak depends on the atomic number of the metal. potential, why most

[7 marks]

1.7) ^{206}Pb is the stable isotope at the end of the ^{238}U decay series. The decay series is dominated by the half life of ^{238}U, which is 4.5×10^9 years. A sample of granite is found to contain 1×10^{-8} kg of ^{238}U and 4.0×10^{-9} kg of ^{206}Pb. Determine the age of the granite.

[7 marks]

1.8) Describe the properties of quarks and describe how composites of quarks form hadrons.

[7 marks]

SECTION B – Answer TWO questions

2)

a) Describe two pieces of evidence for the particle nature of electromagnetic radiation.

[10 marks]

b) In the Bohr model of the hydrogen atom, an electron performs circular motion of radius r around a proton.

i) Using the de Broglie relation, show how wave-particle duality can account for the quantisation of angular momentum $L = nh/2\pi$, where n is an integer.

[4 marks]

ii) Show that the radius of the electron orbit in the Bohr hydrogen atom is quantised.

[6 marks]

iii) Write down expressions for the kinetic and potential energies of the electron. Show that the quantised total energy E_n of the electron is given by

$$E_n = -\frac{m_{\mathrm{e}} e^4}{8\epsilon_0^2 h^2}\frac{1}{n^2} \quad .$$

Determine the wavelength of light produced when an electron in the $n = 4$ energy level drops down to the $n = 2$ level.

[10 marks]

3)

a) What is nuclear fission? Explain why fission seldom occurs spontaneously.

[5 marks]

b) The fission process induced by a low-energy neutron incident on a ^{235}U nucleus is:

$$^{1}\mathrm{n} + ^{235}\mathrm{U} \rightarrow ^{141}\mathrm{Ba} + ^{92}\mathrm{Kr} + 3\,^{1}\mathrm{n} \quad .$$

Describe the mechanism for this reaction.

[6 marks]

The masses of ^{235}U, ^{141}Ba and ^{92}Kr are $235.044\,m_{\mathrm{u}}$, $140.914\,m_{\mathrm{u}}$ and $91.897\,m_{\mathrm{u}}$, respectively. Calculate how much energy, in MeV, is produced in this reaction. In what form is this energy released?

[7 marks]

c) Describe the operation of a fission reactor. Include in your answer a description of how the nuclear reaction is sustained and controlled and how energy is extracted.

[12 marks]

4)

a) How is the molar heat capacity of a gas at constant volume related to the total internal energy U_m of a mole of the gas?

Write down an expression relating U_m to the average energy $\langle\epsilon\rangle$ of each molecule in the gas.

[3 marks]

b) A diatomic molecule can be represented as two equal masses m separated by a spring of length d and spring constant k. In the hydrogen molecule $d = 0.74 \times 10^{-10}$m and $k = 2400\,\mathrm{N\,m^{-1}}$.

i) Explain why there are two rotational degrees of freedom for such diatomic molecules. Write down an expression relating the average rotational energy $\langle\epsilon_{rot}\rangle$ of molecules in the gas to the temperature T.

Show that the rotational energy of a molecule is given by

$$\epsilon_{rot} = \frac{L^2}{md^2} \quad ,$$

where L is the total angular momentum.

The total angular momentum is quantised, with the angular momentum of the lowest rotational mode being given by

$$L = \frac{\sqrt{2}h}{2\pi} \quad .$$

Calculate the energy needed to excite the lowest rotational mode of a hydrogen molecule. Estimate the temperature at which the majority of hydrogen molecules are able to rotate.

[12 marks]

ii) Explain why there are two vibrational degrees of freedom for a diatomic molecule. Write down an expression relating the average vibrational energy $\langle\epsilon_{vib}\rangle$ of molecules in the gas to the temperature T.

The vibrational energy of a molecule is quantised. The energy needed to excite the lowest vibrational mode is related to the vibration frequency ν_{vib} of the molecule by the expression,

$$\epsilon_{vib} = h\nu_{vib} \ , \ \ \nu_{vib} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \quad .$$

Calculate the energy needed to excite the lowest vibrational mode of a hydrogen molecule. Estimate the temperature at which the majority of hydrogen molecules are able to vibrate.

[8 marks]

c) Sketch a fully labelled graph of the variation of the molar heat capacity for hydrogen gas at constant volume as a function of temperature. Explain the form of the graph.

[7 marks]

5) The potential energy $V(r)$ between atoms in a solid can be described by the Lennard-Jones potential:

$$V(r) = \epsilon \left[\left(\frac{a_0}{r} \right)^{12} - 2 \left(\frac{a_0}{r} \right)^{6} \right] \quad ,$$

where r is the separation between atoms.

a) Sketch the form of this potential, indicating on the graph the equilibrium separation a_0 and the binding energy ϵ.

[5 marks]

b) Show that a bond between two atoms of mass m in equilibrium can be considered as a spring with spring constant k, given by the expression:

$$k = \frac{72\epsilon}{a_0^2}$$

[10 marks]

c) Hence show that the Einstein frequency ν of vibration of atoms at low temperatures is given by the expression:

$$\nu = \frac{1}{2\pi} \sqrt{\frac{72n\epsilon}{3a_0^2 m}} \quad ,$$

where n is the number of nearest neighbours of each atom.

[9 marks]

d) Explain, with reference to the form of the Lennard-Jones potential, why the solid will expand when it is heated.

[6 marks]